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ALTERATION REPRESENTATION IN THE PROCESS OF TRANSLATION GRAPHIC TO GRAPHIC

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Abstract

Purpose of the study: This research aims to analyze the alteration representation in the process of translation graphic to graphic where students were asked to solve derivative and integral graphics problems.

Methodology: This research used a pseudo-experimental study with Pretest-posttest nonequivalent Group Design**.** This research was conducted on 24 mathematics education students. Students were asked to solve derivative and integral graphics problems. Some samples of students were interviewed to determine the translation process.

Main Findings: The results of the study showed that the subject used two different methods to carry out the translation process. Thus, all students perform the process of translating the graphic representation into graphs with two different methods, namely the interval and the symbolic.

Applications of this study: The implications of this study indicated that the translation process can help students solve problems, especially problems with graphs and algebra, also revealed activity in the translation process different from that of other researchers before.

Novelty/Originality of this study: Researchers express a new activity to the translation by calling the conversion Intermediary In the stages of Preliminary Coordination (PC) and Objective Construction (CT), S1 and S2 show different forms.

Keywords: *Translation Activities of PC and CT, Representation, Graphic, Derivative, Integral.*

INTRODUCTION

The success of learning mathematics depends on how the understanding and construction of relationships between mathematical concepts (Ramful, 2009, 2014; Cangelosi, Madrid, Cooper, Olson, & Hartter, 2013; Flanders, 2015). Van de Walle (2007) states that mathematical comprehension is a measure of the quality and quantity of connections of new mathematical ideas with existing ideas. The more connections students make to the network of ideas, the better the understanding. If the many opportunities are given to students to think, develop and test the ideas that arise, the more likely they are to build and integrate the network into a rich concept to develop a deeper understanding. Representation as a useful tool to build and communicate both ideas and mathematical understanding (Hjalmarson, 2007; NCTM, 2000; Pape & Tchoshanow, 2001; Tripathi, 2013; Abrahamson, 2006; Goldin & Shteingold, 2001; Kilpatrick, Swafford, & Findell, 2001; Lesh & Post, 1987).

LITERATURE REVIEW

"Representation is a configuration that can represent something else in various ways" (Goldin, 2002). People develop representations to interpret and remember their experiences in an effort to understand the world. Bruner (1966) call appropriate types of enactive iconic and symbolic representation. Most researchers agree that these three types of representation are important for human understanding. Other researchers have reduced three types for two categories (Clark & Paivio, 1991; Marzano, 2004; Marzano, Pickering, & Pollock, 2001). Lesh et al., (1987) found five types of representations to represent mathematical ideas: real-world situations, manipulative models, images, oral/written language and written symbols. In various forms of representation, if students have the ability to move between these representations, they can improve understanding and retention of mathematical concepts.

The ability to move between multiple representations is a characteristic of the definition of mathematical comprehension (Tall, 1992). Janvier (1987) refers to the act of transition from one representation to another representation as trans results. Adu-Gyamfi, Stiff, & Bossé (2012)state that translation is a cognitive process to convert information from one form of representation to another. Lesh et al., (1987) describe three actions, all of which require advanced translation between various representations, which are evidence of student understanding. He said that "part of what we mean when we say that 'understanding' students are that: (1) you can recognize the idea embedded in a variety of different systems of representation qualitatively, (2) which can be flexible to manipulate ideas in certain systems of representation, and (3) can accurately translate ideas from one system to another."

Translation activities from one representation to another are very important in the learning process to express mathematical ideas (Duval, 2006; Bal, 2015; Bossé, Adu-Gyamfi, & Cheetham, 2011). In addition, NCTM (2000) suggests that students "choose, apply and translate mathematical representations to solve problems". However, reality shows that the ability to translate representations between verbal, tables, graphical and symbolic relationships in mathematical relationships is still low (Gagatsis & Shiakalli, 2004). Previous research related to the translation of mathematical representations has been carried out at all levels of education, both in secondary education (Çelik & Sağlam-Arslan, 2012; Bossé, Adu-Gyamfi, & Chandler, 2014; Rahmawati, Purwanto, Subanji, Hidayanto, & Anwar, 2017) and higher (İpek & Okumuş, 2012; Bal, 2015). İpek & Okumuş, (2012) to conduct research on future primary school teachers in the use of representations that can not translate between representations. Çelik & Sağlam-Arslan, (2012)observed that pre-primary school teacher service in determining the capacity of translation between verbal representations, tables, graphs, and physical context. In their study, they found that students have not been able to translate from a physical context to a graphic instead of making a verbal translation to a graphic. While Bal (2015), in a study conducted with 134 primary school teachers prior to service in state universities in Turkey, they found that they generally were not successful in translating verbal representations to other representations. The teachers reported that several representations help to better understand the problems and produce some solutions. Bal suggests that further research should examine the translation process between representations and various forms of representation in problem-solving. Bossé, Adu-Gyamfi, & Chandler (2014) who investigated high school students aged 15 to 17 found that there were four activities carried out by students to translate from a graphic to a symbolic one. These activities consist of *unpacking the source*, *preliminary coordination, constructing the objectives* and *determining the equivalence*. Bossé, Adu-Gyamfi, & Chandler (2014) suggest that other studies translate mathematical representations that are different from graphic to symbolic ones and it is necessary to carry out research to examine in more detail the translation process. Rahmawati, Purwanto, Subanji, Hidayanto, & Anwar, (2017) have examined the process of translating verbal representation into graphs in mathematics education students. The results of the study show that students can do well the process of translating verbal representation to the graph. The process of translating verbal representation into graphic representation requires more than one translation process. This process is through intermediaries of several other representations such as symbolic, schematic, equation, numerical. This investigation is limited only to the verbal representation of the graph. The limitations in this study have the potential to be studied in future research.

In relation to previous research, the results of this study will describe the process of translating the graphic representation to the graphic representation k. It is expected that the results of this study will be used to reflect and evaluate the translation process in the form of representation as a consideration in preparing for correct learning.

METHOD

This research was conducted on 24 mathematics education students. Subjects were asked to solve derivative and integral graphics problems (Figure 1) adapted from Hong & Thomas (2015). Some samples of students were interviewed to determine the translation process. The translation process of the subject was observed based on the translation activity of Bossé, Adu-Gyamfi, & Chandler (2014).

Problems

The subject is asked to become adequate clear methods of solution to any problem. The problem asked the subject to show the translation between the graphic representation and the graph. Problem 1, the subject was asked to make a graph $f'(x)$ when it let the graph of $f(x)$. Problem 2 given a graph of $f'(x)$, then the subject was asked to make a graph $f(x)$. In both cases, there are no specific points in the graph, but given a grid of Cartesian coordinates. This will direct the subject to find the necessary points to solve the problem.

The subjects in this study were first-year mathematics education students who had taken Calculus 1. The subjects had acquired knowledge of the differentials and integrals since high school and deeper when taking Calculus courses. The researcher selected 8 of the 24 students as subjects according to the method of problem-solving and the subject's will.

Analysis of data

Investigate data in the form of oral and written behavior of the subject when solving the given problem. Oral behavior records in the form of interviews are recorded as a way to clarify written responses to the topic and to answer additional questions designed to assess their knowledge.

Data analysis consists of transcribing the collected data, analyzing and reducing it, collecting each part of the data in a unit that is then coded, analyzing the translation process and drawing conclusions. The analysis of the data is done during the investigation when the data is taken simultaneously or after the data collection.

The responses of the subjects were examined to determine the activity of the subject in terms of the translation activity of how to *obtain the unpacking* (US), *preliminary coordination* (PC), *construction of the objective* (TC) and *determination of equivalence* (DE)). For example, if in response, the subject sees the problem as an origin representation that contains certain information, then the response is coded as EE. UU The activities in other answers are analyzed in the same way.

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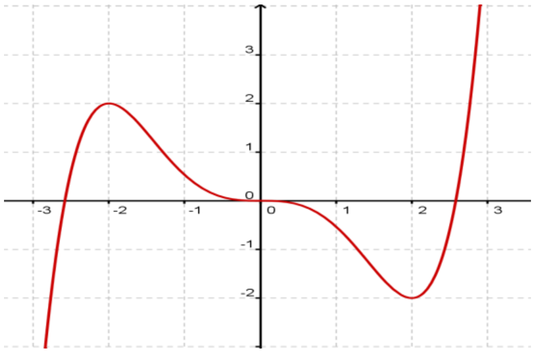
1. *The following figure is a sketch of the function* $f\ (x)$. *Sketch a graphic of the* $f\ '(x)$.



2. *The following figure is a sketch of the function* $f\ '(x)$. *Sketch a graphic of the* $f\ (x)$.

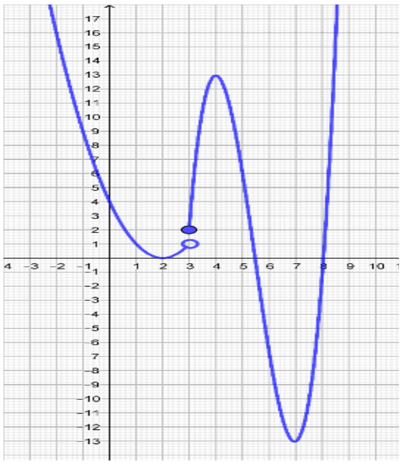


Figure 1: Differential and Integral Graph

Table 1: Categories of translation activity (Michael J Bossé et al., 2014)

Unpacking the source (US)	• Dismantle the information contained in the question. • Identify what is known and ask about the question.
CoordinationPreliminaryC(PC)	• connect the information that has been dismantled in the stage of *unpacking the source* with a concept that has been understood. • prepare information/features that could be used to build the destination representation. • Creating networks and relating ideas between representations.
Building the target (CT)	• transfer the information/information contained in the representation of origin to the destination representation. • Complete the description of the target representation.
Determination of thequivalenceAND(OF)	• verification of the information/information contained in the representation of origin to the destination representation, • Consider ideas and the same in the representation of destination origin and representation.

RESULTS

Table 2 shows the categorization of methods to solve problems in the subject. Of the 24 solutions obtained, 17 (71%) used ascending and descending intervals (representation), 6 (25%) used equation functions (symbolic representation) and 1 (4%) used other methods that were not appropriate (staircase functions) to solve problems number 1. In problem number 2, there were 14 (58%) using up and down intervals, 3 (12.5%) using function equations and 7 (29.5%) not solving problems or using inappropriate methods.

Table 2: Category of settlement method

Problem	Method		
	intervals up and down	symbolic	Another method
No 1	17	6	one
No 2	14	3	7

From the method of solving the topic, two subjects were selected according to the methods of ascending and descending interval and symbolic interval to be interviewed to find out the translation activity.

Subject 1(S1)

The first translation activity is to dismantle the source (US). When observing problem number 1, S1 immediately knows the important information it contains. S1 expresses the verbal repetition when mentioning the command of the question to which the derivative is requested. In addition, S1 indicates that there are peak or stationary points and up and down intervals. This shows that the S1 has carried out activities in the USA. UU., Namely, dismantle the information contained in the question and identify what is known and asked about the question. (Lesh et al., 1987). From the component of the verbal representation, S1 is translated in numerical form in the form of stationary point coordinates and line intervals (interview appointments).

P : *Of the problem number 1, What information can you find?*

S1 : *For the first problem, the problem had to make a graph of f. which is the derivative of f, then I see the stationary/peak points at intervals up and down*

P : *What point / stationary interval/peak do you see?*

S1 : *The stationary point (-2,2), (0,0) (2, -2).From here I can know that the gradient of the tangent line is 0, then the derived graph must intersect or touch the x-axis then the interval (-3.2), (-2.0), (0.2), (2.3) from here I can know that when the graph increases in a certain interval, the graph of the derivative is above the x-axis and vice versa, with the graph descending*

P : *Is there any unnecessary/excessive information?*

S1 : *Do not*

P : *Apaquestion of enough information to answer the questions?*

S1 : *It is not enough, it would be better if the formula of the function were provided so that the derived graph was more appropriate*

A similar activity occurs when S1 solves problem number 2. S1 sees the components (US activity) at top and bottom intervals and stationary points in the representation of the source (graphic). From the interviews, it seems that S1 translates these representations of origin into verbal forms (intervals up and down, stationary points) and numerical (points of coordinates, intervals). From the extract of the interview, it is shown that S1 has linked (PC activity) a network of ideas from verbal and numerical representations made with the concept of minimal minimum graphs and the position of the graph on the axis. (Lesh et al., 1987).

P : *for number 2. Of the problem, what initial information can you know?*

S1 : *Interval up and down the graph f ', stationary point,*

P : *What point / stationary interval / peak do you see?*

S1 : *From the graph there are points that are on the x-axis, (2.0) and (5, 0) which means in the graph f at that point in the maximum/minimum state and then for the interval (-no a, 2), (2.4), (4.7), (7, not up to), from this interval we can know that at the time of the graph d on the x axis then the graph f increases and vice versa*

Figure 2 shows that S1 has linked the ideas that belong to the information obtained in the problem. This is made clear by quotations from interviews with S1 previously, S1 has linked the information form a stationary point and the interval of

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up and down (the representation of the source) with a concept that would have had tangent gradient and position of the derived letter in terms of the axis. The activities carried out by S1 are included in the PC activities, that is, they link information that has been dismantled in the *unpacking* stage of *the source* with concepts that have been understood. and creating networks and relations of ideas between representations (Duval, 2006).


pada saat grafik f berada pada titik puncak, grafik f' berada pada sumbu x

pada saat grafik f naik, grafik f' berada di atas sumbu x

Pada saat grafik f turun grafik f' berada di bawah sumbu x

Figure 2: Answer S1 when the problem of preliminary coordination one

S1 then manages the information (source representation) and network ideas to then prepare information/features that could be used to construct the target representation (CT activity). S1 connects verbal components that have been expressed numerically to form the statement in Figure 3. With this, S1 can obtain the properties of the graph to be addressed based on the points and the slope of the line and can help to make graphic images in the next translation activity.

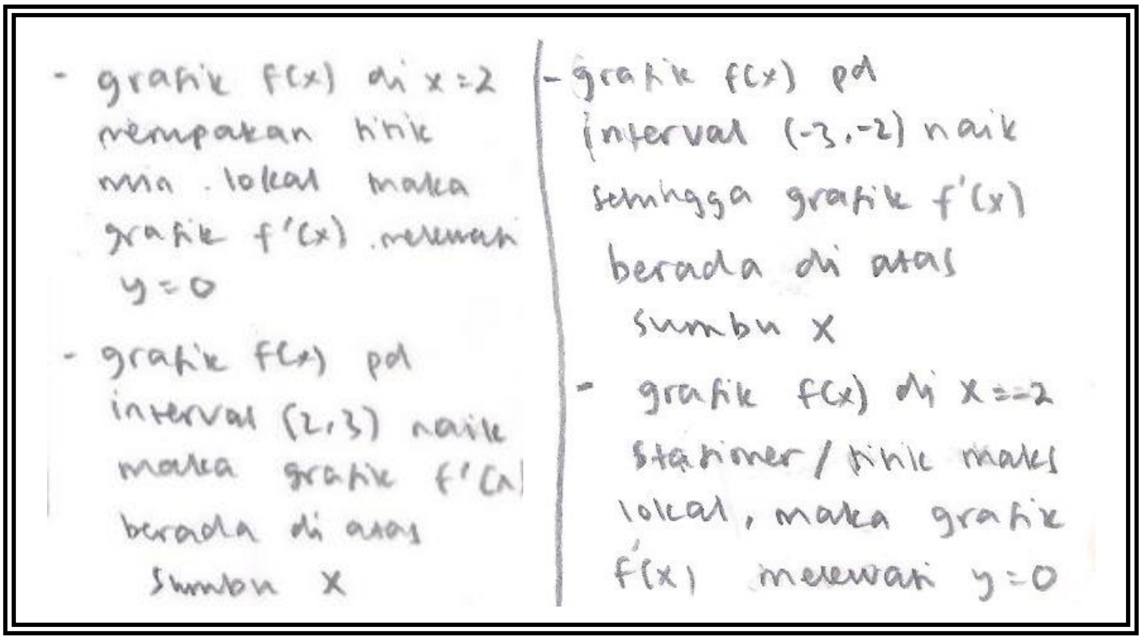
- grafik f(x) di x=2 merupakan titik min. lokal maka grafik f'(x) melewati y=0
- grafik f(x) pd interval (2,3) naik maka grafik f'(x) berada di atas sumbu X
- grafik f(x) pd interval (-3,-2) naik sehingga grafik f'(x) berada di atas sumbu X
- grafik f(x) di x=-2 stasioner / titik maks lokal, maka grafik f'(x) melewati y=0

Figure 3: Answer S1 to build objective problem 1

In addition, S1 constructs a target representation (CT) in the form of a graph based on the relationship of the ideas that have been made. S1 transfers the information/information contained in the representation of origin to the destination representation (Lesh et al., 1987). S1 made graph according to the information on stationary points in $x = 2$, $x = -2$, and $x = 0$ that would cause the target graph to be on the axis. S1 specified that the interval information increases in $(2,3)$, $(-3,-2)$then the graph of $f'(x)$on the axis (Based on interview appointments of S1).

P : *After using the concept of the graph up and down, the slope of the tangent line, how do you draw the graph f '(x)?*

S1 : *Drawing according to the interval, and the stationary point that is known to know the derivative will be on the x-axis*

When the CT's activity occurred in problem 2, S1 stated that the method used as opposed to the method used in problem 1 (interview appointment of S1). Figure 4 shows S1 transferring information in the form of stationary points and intervals. (representation of the source) to the graph $f'(x)$as an objective representation (Lesh et al., 1987).

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P : *Then how do you do it?*

S1 : *number 2, opposite to number 1, wherein the graph f 'already exists for the intervals, when the graph f' is on the x-axis, it means that at the point where the graph f 'crosses the x-axis, the point in the graph f is the peak point, as we already know that f 'is the slope of the tangent f, so we can know that when f' is on the x-axis, the graph f is rising, whereas when f 'is below of the x-axis, the graph f is being below*

P : *You said number 2 opposite number 1, what do you mean?*

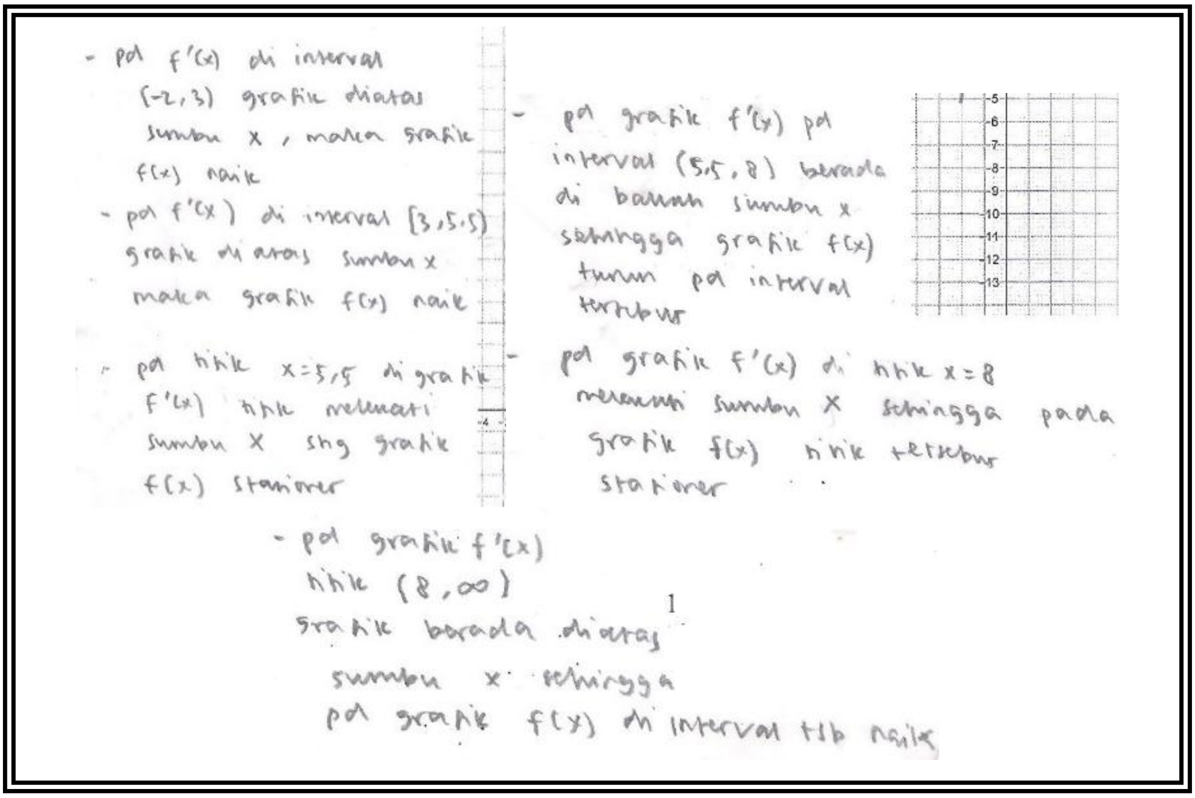
S1 : *From the graph f 'we can draw the graph f when the graph f' is on the x-axis, it means that the graph f is at the stationary point*

P : *So with number 1, it has to do, huh?*

S1 : *Yes, there is a connection, because the concept that I use is the same*

P : *To draw graphics, how to place the points?*

S1 : *I just put it if, for example, d graph f 'cuts / is on the x-axis. Then, the graph f is at the local maximum/minimum point. Like mbak, if for the location I only guess aja mbak*



- pd f'(x) di interval (-2,3) grafik diatas sumbu x, maka grafik f(x) naik
- pd f'(x) di interval (3,5.5) grafik di atas sumbu x maka grafik f(x) naik
- pd titik x=5,5 di grafik f'(x) titik melewati sumbu X shg grafik f(x) stasioner
- pd grafik f'(x) pd interval (5,5, 8) berada di bawah sumbu x sehingga grafik f(x) turun pd interval tersebut
- pd grafik f'(x) di titik x=8 melewati sumbu X sehingga pada grafik f(x) titik tersebut stasioner
- pd grafik f'(x) titik (8,∞) grafik berada diatas sumbu x sehingga pd grafik f(x) di interval tsb naik

Figure 4: Answer S1 when constructing objective problem 2

In the fourth activity, DE, S1 does not revise problems 1 and 2, but when interviewing the subject, he is sure of the answer in the way he knows it (interview appointment).

P : *How is b checkedtoHwaeveryonecorrect grafikmu?*

S1 : *Due to the limited information of the questions and limited knowledge, I still can not verify if the graphics I have drawn are correct or not, I only make a sketch according to what I know and information about the questions.*

Subject 2 (S2)

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S2 sees the graph of problem number 1 as a representation of odd-degree polynomial functions. In this first activity, S2 directly translates the representation of the source graph into a verbal form without identifying the components of the graph. From the information obtained, the subject is associated with the concept that is held on the ups and downs of the graph and the negative-positive value of $f'(x)$ (appointment of the interview). This shows that S2 performs PC activities by linking information that has been dismantled at the stage of *unpacking the source* with concepts that have been understood and by creating networks and ideas for relationships between representations (Duval, 2006)

P : *From problem number 1, what information can you find?*

S2 : *From the initial graphical form, it seems that the polynomial function is odd,*

P : *Next, what do you do to solve it?*

S2 : *in the calculation already discussed about f (x) going up and down in relation to the value of f '(x) negative-positive, maximum point and inflection point = intersection and allusion on the x-axis, only that at that moment I did not know the maximum point f '(x) (there are 2, which are clearly symmetric on the y-axis)*

Similar activities occur when S 2 solves problem number 2. S 2 immediately sees the representation of the source of the graph as a representation of the second-degree polynomial function. S2 seeks to solve problems by thinking about linking the concept of intermittent graphics (PC activity) with symbolic representations (interview appointments).

P : *for number 2. Of the problem, what initial information can you know?*

S2 : *The way to find the graph is the same, simply inverted. But my explanation is not entirely correct. Initially, I tried to estimate the graph at the time x <3 and x>=3, it seems that the polynomial function is grade 2 (x-2) ² and grade 3, which at the point y = 0 is the factor.*

Next, in Problem 1, S 2 constructs a target representation (CT) by transferring source representation information in the form of verbal representation of a polynomial graph of the odd degree to the target representation (Lesh et al., 1987). In Figure 5, S2 uses an odd-degree polynomial function to obtain the properties of the graph $f'(x)$as a polynomial function of uniform degree with tangent points on the axis. After drawing graphs, S2 does not check (DE) on problem 1.

(1) Grafik f(x) merupakan grafik fungsi polinom dengan derajat ganjil. Maka, grafik f'(x) merupakan fungsi polinom derajat genap dengan x-intercept di (2,0) dan (-2,0) serta menyinggung origin (0,0) Terbuka ke atas.

Figure 5: Answer S2 to build the problem objective one

$f(x) = \frac{1}{3}(x-2)$

(3) untuk $x < 3$, dari grafik bisa diketahui bahwa $f(x) = (x-2)^2, = x^2-4x+4$ dan untuk $x \geq 3$, $f'(x) = (x-3)(x-11/2)(x-8)$ tidak terpenuhi untuk $f'(4) = ((-1,5)(-4) = 6$ karena terdapat pembesaran

Figure 6: The response of S2 when constructing objective problem 2

S2 uses estimates to obtain the factors of the functional equation (CT activity) given the concept of displacement and enlargement of the graph of the function even if it does not work correctly (Figure 6). Next, S2 performs a graphical function equation to verify the critical point (DE) even if the answer is not correct (citation of the interview).

P	:	*So, how do you draw the graph f '(x)?*
S1	:	*Now I just remember that the function can not only be moved, but also raised (vertical-horizontal), so the writing does not apply to the extension. Initially, I found a new graphic image for the equation to ensure the critical point f (x)*

DISCUSSION

On the basis of the answers and interviews in subjects S1 and S2, the results of the study show that after the subject performs the activity of dismantling the source of representation (US) in both problems, the subject changes the form of representation of the representation of origin (graphic) to another form of representation. S1 changes the representation of the font (graphic) to verbal and numerical forms (intervals up and down, coordinate points), while S2 changes directly to the verb form (even / even pair polynomial function) without looking at the graphic components. This shows that there are different activities of the *Unpacking Source* activities described by Bossé et al., (2014). The activity US of Bossé et al., (2014) includes the dismantling of the information contained in the question and the identification of what is known and asked about the question. While in this study, the subject changed the form of representation of origin to other representations directly before associating with the concepts they had. The subject requires an intermediate representation in the translation of the representation of origin to the representation of destiny. This activity can be explained in the construction proposed by Piaget (Dubinsky & McDonald, 2001), namely, the interiorization stage. Internalization is the construction of internal processes to understand the phenomenon that is felt. Duval (2006) uses the term conversion and defines it as the process used to change the representation of records without changing the object represented. Therefore, researchers in this stage express a new activity to the translation by calling *the conversion Intermediary*.

In the stages of *Preliminary Coordination* (PC) and *Objective Construction* (CT), S1 and S2 show different forms. S1 uses the concept of tangent gradients and ascending and descending intervals to determine the characteristics of the desired graph (objective representation). While S2, use function equations and intervals up and down. This is in accordance with the translation activities of PC and CT. explained Bossé et al., (2014). On *determining equivalence* (DE), subject 1 does not perform the activity but believes that the solution is in accordance with the concept he knows. S2 performs the function equation in point checking, but the subject's response to problem 2 does not work correctly. For the findings of this activity, the researchers structure the translation process carried out by S1 according to the translation activities in this study (figure 7).

Annotation:

⟶	Process carried out		Activities that are not done.
	Collection of activities	***TP***	High / stationary point
US	*Unpacking the* activities of *the source*	***MS***	The point is the question.
KV	Conversion activity	***EnNT***	Interval up and down
PC	*Preliminary Coordination* Activities	**In**	Interval
KL	Continuous coordination activities.	***Grd GS***	Tangent gradient
TC	*Building the objective* activities	***standard***	Questionnaire S
OF	*Determination* of *equivalence* activity	***Grf***	Graph
Bc	Read the question	***skm***	Make a scheme
idf	Identify	***sk***	Sketch
center	Connect	***check***	Check it out

CONCLUSION

On the basis of the research carried out, it is concluded that all students perform the process of translating the graphic representation into graphs with two different methods, namely the interval and the symbolic. This is seen when the subject is in *activities of Conversion Intermediary.* Therefore, the translation processes. In this study carried out in five stages: *Unpacking Source*, *ConversionIntermediary*, *Preliminary Coordination*, *Construction of Objectives*, and *Determine equivalence* according to NCTM, the use of various forms of representation can support understanding of mathematical concepts, in this case, the translation between representations, making it easier to communicate mathematics because of understanding the relationships between mathematical concepts. Therefore, the use of translation between forms of representation needs to be mastered by students so that they can solve various problems with various forms of representation. However, in this study, S1 and S2 used a similar method in solving problems 1 and 2. This is due to the limited experience of students in knowing various forms of representation.

LIMITATIONS AND STUDY FORWARD

Given the conceptual relevance of problems 1 and 2, this study has not yet paid more attention to the opinions of the subject. The translation process of the subject's representation in managing relationships and changes between forms of representation needs to be studied further. The limitations of this study have the potential to be studied in future research.

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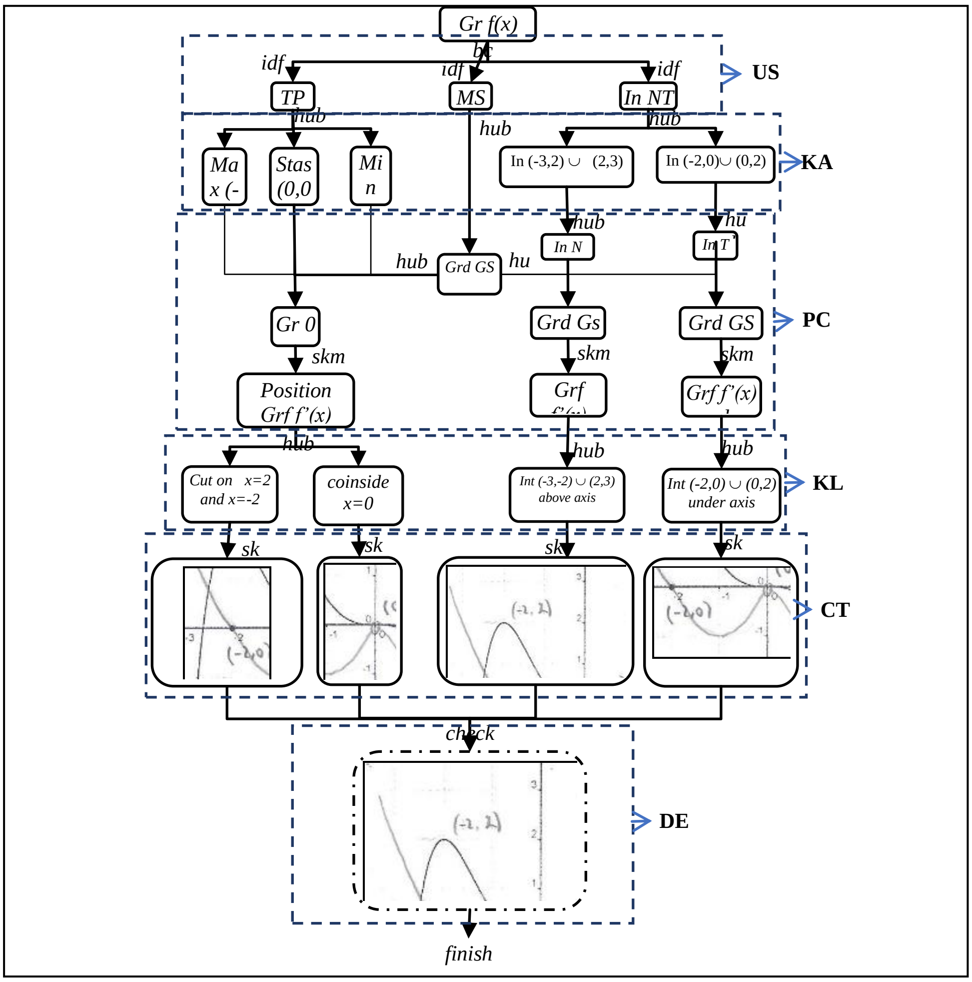

Figure 7: The translation process according to the translation activities in this study

