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STAGES IN PARTIAL FUNCTIONAL THINKING IN THE FORM OF LINEAR FUNCTIONS: APOS THEORY

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Abstract

Purpose of the study: The purpose of this study is to describe students' partial functional thinking processes in solving mathematical problems based on APOS Theory. The problem in this study was formulated into the question, what are the stages of students' partial functional thinking in solving mathematical problems based on APOS Theory?.

Methodology: This study was conducted by with 44 students from the Department of Mathematics Education. The subjects of this study were asked to solve mathematical problems developed from (Wilkie, 2014). Then some of them were interviewed to learn their functional thinking processes. The subjects' partial functional thinking processes were analyzed using APOS theory.

Main Findings: The results showed that, based on APOS theory, the students' partial functional thinking consisted of several stages: 1) identifying the problem, 2) organizing the data, 3) determining the recursive patterns, 4) determining the covariational relationships, 5) generalizing the relationships between variations in quantities (correspondence), and 6) re-checking the generalization results. In this case, the students generalized the relationships between variations in the form of functions done partially using the arithmetic formula $U_n = a + (n-1)b$.

Applications of this study: The findings of this study can help teachers understand the stages in students' thinking processes in solving problems about functions and the difficulty faced by the students in understanding the functions.

Novelty/Originality of this study: The researchers identified stages in students' partial functional thinking in solving mathematical problems in the form of functions based on APOS Theory.

Keywords: *Functional Thinking, Partial, Linear Functions, Problem Solving, APOS.*

INTRODUCTION

Functional thinking is an important aspect of mathematics learning at school (Stephens, et al, 2011; Tanişli, 2011; Warren, et al, 2006). Functional thinking is defined as representational thinking that focuses on the relationship between two (or more) variations of Smith's quantity (Markworth, 2010). This is in line with the statement of (M. Blanton et al., 2015), stating that functional thinking involves the generalization of relationships between covariant quantity, reasoning, and representing these relationships through natural language, algebraic notation (symbol), table, and graph. The benefits of functional thinking are that it: 1) facilitates students in understanding algebra and functions; 2) can be used as an alternative way of thinking in generalizing the relationship between quantity variations; 3) can be used to develop students' reasoning ability; and 4) can be used as a basic competency to support the success in calculus, advanced mathematics, or science (Tanişli, 2011).

According to (Stephens et al., 2011), functional thinking can be integrated into learning and curriculum. The 2013 curriculum requires students to 1) Understand patterns and use them to guess and make generalizations (conclusion), 2) Use patterns and generalization to solve problems, and 3) conduct experiments to find an empirical opportunity of real problems and present them in table and graph. NCTM (2000) also states that students in school must be able to: 1) understand patterns, relationships, and functions; 2) Represent and analyze mathematical situations and use algebraic Symbol structures; 3) Use mathematical models to represent and understand quantitative relationships, and 4) analyze changes in various contexts. Thus functional thinking is very important to be implemented in mathematics learning in order to fulfill the demands of the curriculum. Blanton et al., (2016) provide examples of functional thinking tasks outlined in the following table (Table 1).

Table 1: Example of Functional Thinking Tasks

Example of Functional Thinking Tasks	Function Type	Explanation
Cutting rope: the relationship between the number of cutting rope and the number of resulting cutting rope.	$y = x + 1$	x = number of cutting ropes y = number of resulting cutting ropes
Candy box: the relationship between the number of Jhon's candy and Mary's candy if John and Mary have a similar number of candy, but Mary has one more candy inside the box.	$y = x + 1$	x = number of Jhon's candy y = number of Mary's candy

Age difference: the relationship betweenBudi's age and Nana's age if Budi is 2 years younger than Nana.	$y = x + 2$	x = Budi's age y = Nana's age
Brady's birthday party: the relationship between the number of square tables and the number of people sitting on the tables if the tables are merged side by side with the condition that no one sits at the end, and only one person sits on every 2 sides of the table.	$y = x + x$	x = number of the square table y = number of people sitting on the table

According to the example of the functional thinking tasks above, it is explained that there is a relationship between two quantities which are then generalized into a form of an appropriate function.

Smith (Stephens et al., 2011 & Tanişli, 2011) mentions three stages in functional thinking that are: 1) recursive patterning which means looking for variations or patterns of variation in a set of values of the variable, so that certain values can be obtained based on previous values, 2) covariational thinking focuses on analyzing two variations of quantity simultaneously and understanding that change is an explicit and dynamic part of the function description (for example, "as x increases 1, y increases 3"), and 3) correspondence relationship is based on identifying the correlation between variables (e.g., "y is 3 times x plus 2"). Furthermore, Blanton et al., (2015) develop these stages into 1) generalizing linear data and organizing them in a function table; 2) identifying recursive patterns and describing them in words, using patterns for predicting precise data; 3) identifying covariational relationships and describing them in words; 4) identifying the rules of function and describing them in words and variables, and 5) using function rules to widely predict function values. This study examined the following stages in functional thinking: 1) identifying the problem, 2) organizing the data, 3) determining the recursive patterns, 4) determining the covariational relationships, 5) generalizing the relationships between variations in quantities (correspondence), and 6) re-checking the generalization results.

LITERATURE REVIEW

Functional thinking in mathematics learning has been widely studied. For instance, Stephens, et al (2016); Blanton & Kaput (2004); Brizuela, et al (2015); Blanton, et al (2015); Blanton, et al (2016); Muir & Livy (2015); Tanişli, 2011; Warren (2012); Warren, et al (2006); Warren & Cooper (2005); Wilkie, (2014) conducted research on Elementary School students. The results showed that the students were able to understand the relationship between quantity variations and begin to think functionally. Blanton & Kaput (2005); Doorman, et al (2012); Stephens, et al (2017); Stephens, et al (2017); Warren, et al (2006); Wilkie (2004, 2015); Wilkie & Clarke (2015, 2016) design learning that can improve students' functional thinking. Then, Mceldoon, 2010 develops an assessment to measure the ability of the elementary school students to think functionally, especially to find the rules of correspondence in the function table. Allday (2017) conducts research on student behavior in functional thinking that can help the teacher make decisions in determining better interventions. However, it has not examined the students' functional thinking processes in solving mathematical problems portrayed using APOS Theory (Action, Process, Objects, and Schemes).

The purpose of this study is to describe students' partial functional thinking processes in solving mathematical problems based on APOS theory. The problem in this study was formulated into the question, "what are the stages of students' partial functional thinking in solving mathematical problems based on APOS Theory?.

METHODOLOGY

This study used an exploratory qualitative research design and involved the 4th and 6th-semester students from the Department of Mathematics Education at the Universitas Islam Negeri (UIN) Sultan Syarif Kasim, Riau. There were 44 students (24 students from semester 6 and 20 students from semester 4) participating in a think-aloud test. Every test taker had to complete the test individually. The students' answer sheets were evaluated by the researchers. The results showed that 16 students submitted the correct answer, and 28 students had the answer wrong (the students made mistakes when generalizing the relationship between quantity variations). This study focused on describing the functional thinking process of the students who were able to provide the correct solution to the problem. The results of the student answer sheet's evaluation and the results of the think-aloud test revealed some functional thinking stages that had not appeared in the classroom and thus required clarification. Thus, interviews were conducted to explore and clarify these issues. The interviews were conducted to 16 students who consisted of 4 students from semester 4 and 12 students from semester 6. The interview data were then analyzed accordingly. Based on the analyses, it was found that 11 students who consisted of 4 students from semester 4 and 7 students from semester 6 performed a partial functional thinking process. Out of the 11 students, two representatives were selected. The triangulation method was conducted to analyze the students' functional thinking process. The data obtained from the think-aloud test, students answer sheet evaluation, and interviews were compared to each other. The questions given to the students were developed based on Wilkie (2014) research. These questions are presented below:

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Problem:
Pay attention to the mosaic patterns of the two-dimensional figures which contain decagons, triangles, and rectangles in figure 1, figure 2 and figure 3 below (these mosaic patterns formed with the same pattern until figure *n*).

figure 1 figure 2 figure 3

Determine some possible relationships between the two-dimensional figures in figure 1, figure 2, figure 3 until figure *n*. Then find the standard formula of these relationships!

These questions have been validated by mathematicians and experts in mathematics education.

RESULTS/FINDINGS

The results of the analysis of the think-aloud test, interviews, and student answer sheet evaluation, the students' partial functional thinking process can be described as follows:

Identifying the Problem

In identifying the problem, subject S1 reads the information on the test sheet. Next, the subject observes figure 1, figure 2, and figure 3 containing two-dimensional figures in sequence, 4 triangles, 6 rectangles, and 1 decagon in figure 1; the second figure contained 7 triangles, 11 rectangles, and 2 decagons; while figure 3 contained 10 triangles, 16 rectangles, and 3 decagons. This was supported by the result of the think-aloud test (S1) presented below:

S1: "In the first figure, there are 4 triangles and 6 rectangles, then there is a decagon. Mm (thinking), this is a quadrilateral. In the second figure, there are 7 triangles, 11 rectangles, and 2 decagons. In figure 3, there are 10 triangles, 16 squares, and 3 decagons".

Similarly, subject S2 started the identification of the problem by reading the information on the test sheet, then observed the number of triangles, rectangles, and decagons contained in figure 1, figure 2, and figure 3 in sequence. The subject found that there were 4 triangles in figure 1, 7 triangles in figure 2, and 10 triangles in figure 3. Besides, figure 1, 2, and 3 contained 6, 11, and 16 rectangles, respectively. There was 1 decagon in figure 1, 2 decagons in figure 2, and 3 decagons in figure 3. This is followingS1's think-aloud data presented as follows.

S2: "The types of the rectangle, triangle, and decagon are known figure 1 ... figure 1 contains 4 Eee triangles, figure 2 has 7, figure 3 (while counting 1, 2, 3, 4, 5, 6, 7, 8, 9, 10) contains 10. Rectangles in figure 1 are 6, in figure 2 (while counting 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. 11) are 11, and in figure 1 -3 (while counting 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16) are 16. There is one decagon in figure 1, figure 2 contains 2 decagons, and figure 3 contains 3 decagons".

Organizing the Data

Subject S1 organized the data by making a list and grouping triangles, rectangles, and decagons in figure 1, figure 2, and figure 3. The statement is following the work of subject S1 in organizing the data presented in Figure 1.

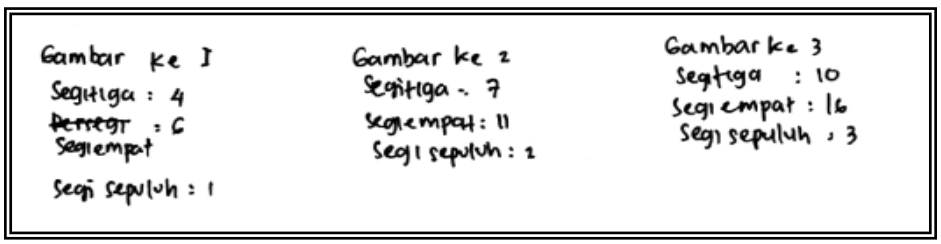

Figure 1: Work Result of Subject S1 in Organizing the Data

Subject S2 also organized the data by making a list and grouping the shapes in each figure, which are triangles, rectangles, and decagons in figure 1, figure 2, and figure 3. This statement is following the work of subject S2 in organizing the data presented in Figure 2.

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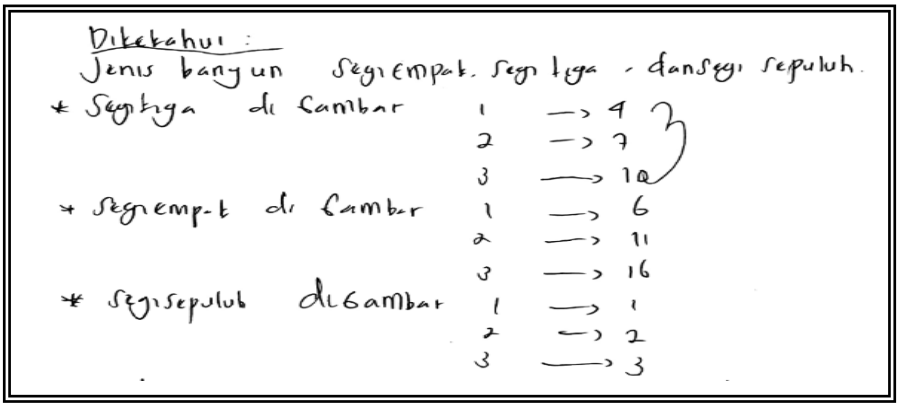

Figure 2: Work Result of Subject S2 in Organizing the Data

Determining the Recursive Pattern

Subjects S1 and S2 explained the pattern of the triangles (symbols) in figure 1, figure 2, figure 3 in sequence by writing the number of the triangles: 4, 7, 10, then looked for differences using the formula $b = U_n - U_{n-1}$, in which $b = 7 - 4$so that $b = 3$. The pattern of the rectangles was obtained from the number of the rectangles in figure 1, 2, and 3, that are 6, 11, 16; hence, $b = U_n - U_{n-1}$, in which $b = 11 - 6$ so that $b = 5$. The pattern of the decagons was obtained by calculating the number of the decagons in the figures, then $= U_n - U_{n-1}$, in which $b = 2 - 1$so that $b = 1$. This is consistent with the subject's work result in determining the recursive pattern in Figure 3.

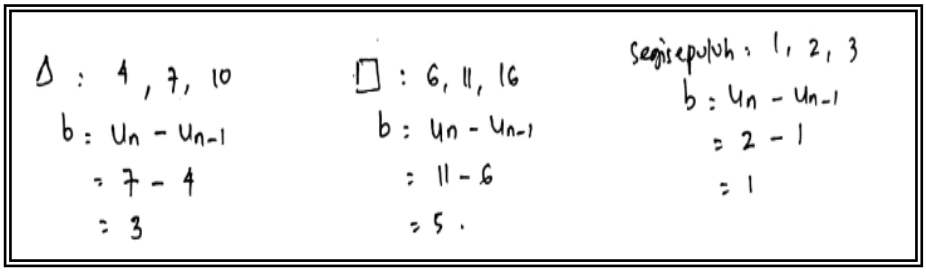

Figure 3: Work Results of Subjects 1 and 2 in Determining the Recursive Patterns of the figures

Determining the Covariational Relationship

Subjects S1 and S2 determined the change in value between the location of an item with the item itself. If the location of the triangles changed in value of 1, then the triangles changed by 3, if the location of the rectangles changed in value of 1, then the rectangles changed by 5, if the location of the decagons changed by 1 then the decagons changed in value of 1.

Generalizing the Relationships Between Quantity Variations

Subject S1 and S2 used the arithmetic formula $U_n = a + (n-1)b$ to determine the nth term. Therefore, it was obtained that U_n for the triangle was $U_n = 3n + 11$; $U_n = 5n + 1$ for the rectangle; while U_n for the decagon was $U_n = n$. The following is the subject's think-aloud data.

> *S1: "After getting the difference, we can find the nth term formula for triangles, U_n for triangles, as we know that the former formula is $U_n = a + (n-1)b$, the value of a is 4 and then added with $(n-1)$ then multiplied with b, as the consequence, the difference is 3, then $4 + 3n - 3$ equals to $3n + 1$. U_n the formula for the rectangle is $U_n = a + (n-1)b$, then the value of a is 6, then added $(n-1)$, then multiplied with b, the value of b is 5, those which equal to $6 + 5n - 5$, so $U_n = 5n + 1$. U_n the formula for the decagons is $U_n = a + (n-1)b$, the value of a and b are the same, which is 1, as the consequence, the formula is $1 + n - 1$, so $U_n = n$ ". So here the general formula for the relationship is: U_n the formula for a triangle is$U_n = 3n + 1$, Un formula for the rectangle is $U_n = 5n + 1$, Un formula for decagon is $U_n = n$ ".*

This is reinforced by the results of the subject's answer sheets in generalizing the relationship between quantities presented in Figure 4.

Re-checking the Generalization Results

Subject S1 and S2 re-checked the results of generalizing the relationship between quantities and they believed that the resulting formulas have been correct. The following are the excerpts of the interviews conducted to subject S1 and S2.

Humanities & Social Sciences Reviews
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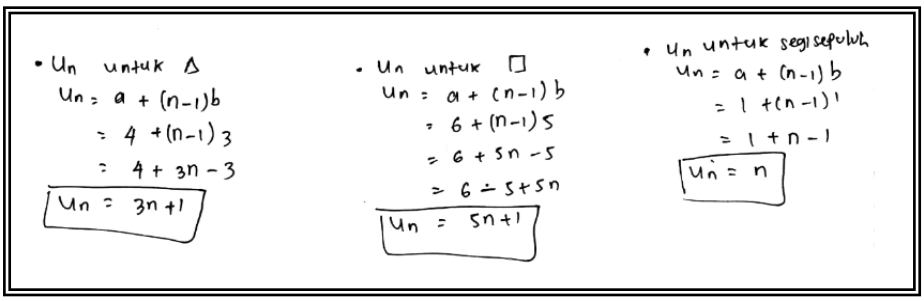

Figure 4: Subject's Generalization Process

P: Ok, then, is that the general formula?

S1: Uhm ... (S1 checks back while thinking), yeah that's right, Mrs...

P: Fine ..., S2, from the general formula, obtained, is it correct?

S2: Uhm... Mrs, So,for triangles if $n = 1$, *then* $U_1 = 3.1 + 1 = 4$, *if* $n = 2$ *then* $U_2 = 7$, *if* $n = 3$ *then* $U_3 = 10$ *until* U_n. *For thent value, if* $n = 1$, *then* $U_1 = 5.1 + 1 = 6$, *if* $n = 2$ *then* $U_2 = 11$, *if* $n = 3$ *then* $U_3 = 16$ *until* U_n. *For tenth value, if* $n = 1$, *then* $U_1 = 1$, *if* $n = 2$ *then* $U_2 = 2$,*if* $n = 3$ *then* $U_3 = 3$ *until* U_n.

This is reinforced by the conclusion made by subject S1 in Figure 5.

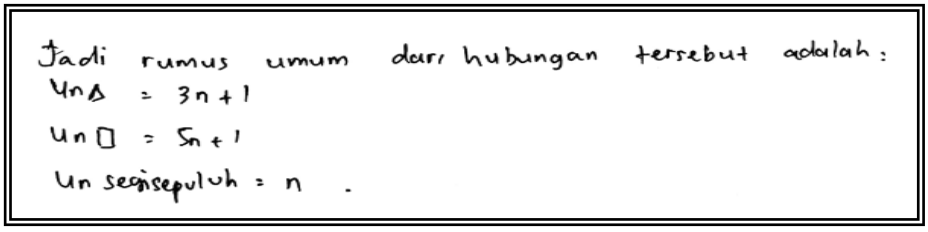

Figure 5: Work Result of the Subject in Making a Conclusion

In general, students' partial functional thinking processes in solving mathematical problems based on APOS theory are presented in figure 6.

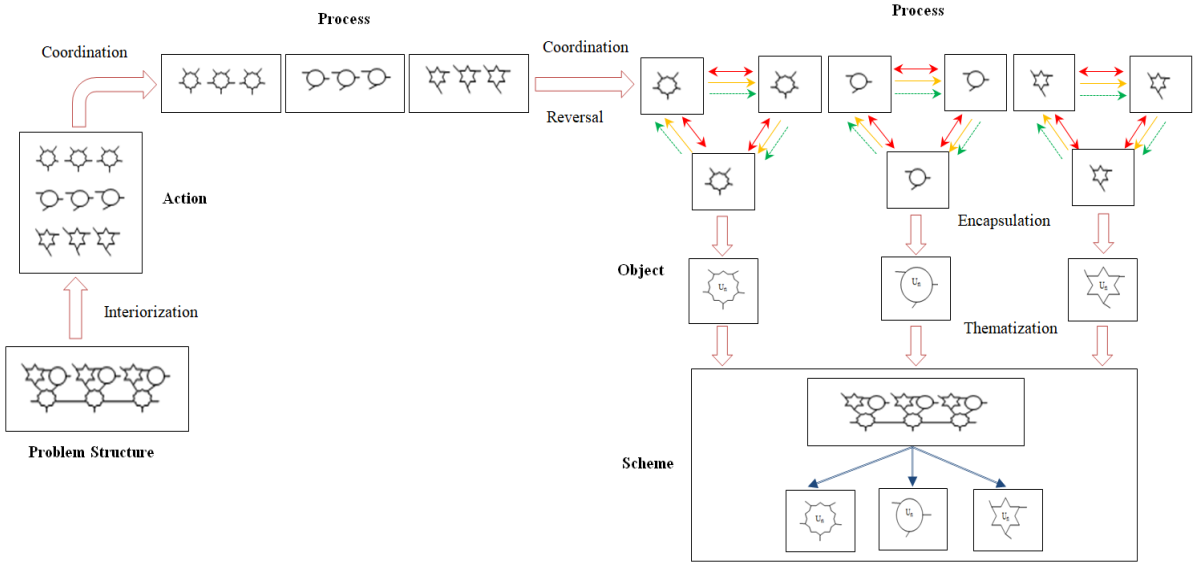

Figure 6: Partial Functional Thinking Process Based on APOS Theory

Remarks

	Decagon	⟷	Recursive pattern
	Triangle	U_n	$U_n = n$
	Rectangle	U_n	$U_n = 3n + 1$

Humanities & Social Sciences Reviews
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→	Covariational relationship		$U_n = 5n + 1$
⇢	$U_n = a + (n-1)b$	⇨	Switch to another mental structure
→	Resulting process		

DISCUSSION/ANALYSIS

Thus, from the analysis of the data above, many stages in a partial functional thinking process are obtained based on the APOS theory carried out by the students, which can be seen in Table 2.

Table 2: Stages in Partial Functional Thinking Processes Based on APOS Theory

Stages of the partial functional thinking process	Student Activities	Mental Mechanism	Mental Structure
Identifying the problem	• Understand and observe figure 1, figure 2, and figure 3 • Represent it numerically	Interiorization	Action
Organizing the data	• Create a list and group the two-dimensional figures found in every figure	Coordination	Process
Determining the recursive Pattern	• Determine the number of two-dimensional figures found in every figure • Use the formula $b = U_2 - U_1$ • Represent with algebra	Coordination Reversal	
Covariational relationship	• Determine the value change of the relationship between quantity variation on a number, which is by determining value change based on the location of the item by item • Represent it verbally		
Generalizing the relationship between the quantities	• Generalize the relationship between quantity variations in the form of functions performed partially • Use the formula $U_n = a + (n-1)b$ • Represent it in algebraic form	Encapsulation	Object
Re-checking the Generalization Results	• Represent it verbally	Thematization	Scheme

Based on Table 2, the initial step taken by the subjects in solving the problem is reading the information on the test sheet. Next, the subjects discuss the problem by discussing the given problem. Observing a certain case is one of the activities in sentence processing to resolve a problem. This is supported by research by (Canadas & Castro, 2007; Cañadas, et al, 2007; Polya, 1973; Reid & Jniversitv; Sutarto, et al, 2016; Yuniati, 2018) which states that involving cases in inductive evaluation processes was carried out on certain cases of the problems raised. Thus, the functional thinking process is included in the inductive evaluation process. Then, the subjects count the objects that match the same shape and color. Data 1, data 2, and data 3 are obtained from grouping the objects. To organize the data, the subjects create a list. In identifying a problem and organizing data, the mental structure that emerges is the Action. This is consistent with the opinion of (Dubinsky, 2001) which states that actions are carried out through physical or mental manipulation that involves the transformation of objects created by external stimuli. External stimuli consist of cognitive objects that have been constructed beforehand in an individual's mind through learning experiences. Mental activity that arises in this activity is called interiorization. This is following the opinion of (Dubinsky, 2001) which states that an individual does the interiorization of actions by repeating and reflecting actions in his mind, so he can translate and explain the transformation process in detail.

The next activity conducted by the subjects is to create a pattern from data 1, data 2, and data 3. The pattern is a recursive pattern obtained inductively using the formula $b = U_2 - U_1$. This is following the opinion of (Pinto & Cañadas, 2012; A. C. Stephens et al., 2011; Tanişli, 2011) who states that determining a recursive pattern can be done by looking for variations or patterns of variation according to the values of variables so that certain values can be obtained through previous values. The recursive patterns of data 1, data 2, and data 3 are made as the benchmark to determine the

Humanities & Social Sciences Reviews
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values of the relationships between variations in quantities (covariational relationships), namely changes in value that occur between items with the items themselves. This is following the opinion of (Wilkie, 2014) which states that covariational relationships in a sequence of numbers occur between the location of the items with the items themselves. The mental structure that arises in this activity is the Process, while the mental mechanisms that occur are coordination and reversal. According to (Dubinsky et al., 2005) coordination is a mental transition in coordinating interiorized actions. Coordination is used to construct a new process. Two or more processes can be coordinated to create a new process. Reversal is an activity to trace the knowledge that was previously owned to construct a new concept.

The next activity is that the subjects generalize the relationship between variations (correspondence) fully consisting of generalizing data 1, generalizing data 2, and generalizing data 3 using the formula of arithmetic sequence namely $U_n = a + (n-1)\,b$. The results of the generalization are represented by using algebraic representations. This is supported by the research findings of (Yuniati, et, al 2019) which state that students use algebraic representations in generalizing the relationship between income. Algebraic representation is the most dominant representation used by students because they learn it from the teachers. Thus, the partial functional thinking process occurring at this stage is a mental activity in generalizing the relationship between variations in the form of functions carried out partially on the variations in the amount given. The mental structure that arises in this activity is the Object, while the mental mechanism that arises is encapsulation. According to (Dubinsky, 2001) an individual is said to conduct encapsulation if he has realized the process as a totality and realized that the action should be carried out in that process.

The final activity in partial functional thinking is to re-check the results of the generalization process and to believe that the resulting formula is correct. The mental structure that arises in this activity is the Schema. Schema is a collection of mental structures of action, processes, objects, and other schemes and combined to form the totality in understanding a concept that is being studied (Dubinsky et al., 2005; Dubinsky & Michael A. McDonald, 2008). In general, the students' partial functional thinking processes are presented in figure 7.

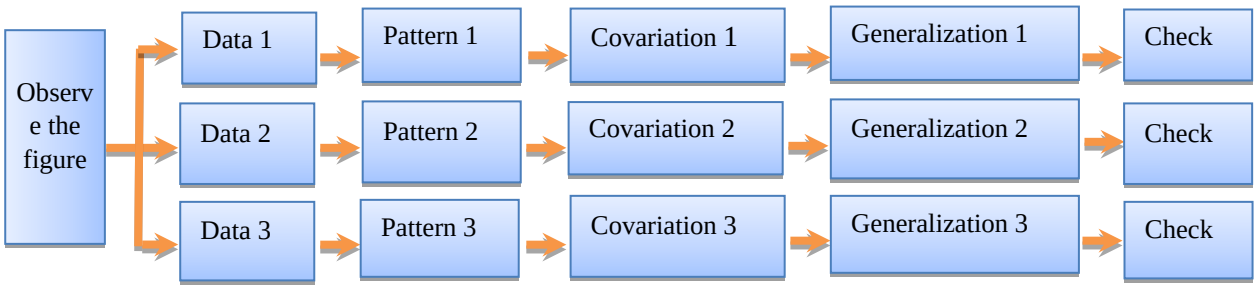

Figure 7: Partial Functional Thinking Process

CONCLUSION

The partial functional thinking process is a mental activity conducted to generalize the relationship between quantity variations in the form of functions carried out partially on the given quantity variations. In this study, the students' partial functional thinking process in solving a problem based on APOS theory consists of six stages through which they are: 1) identifying the problem, 2) organizing the data, 3) determining the recursive patterns, 4) determining the covariational relationships, 5) generalizing the relationships between quantities, and 6) re-checking the generalization results. All stages of partial functional thinking are done well by the students. The students also generalize the relationship between quantity variations partially. The findings of this study provide some insights into mathematics teachers' knowledge of students' thinking processes in function problem solving. The results of this study also help the teachers identify the difficulty faced by the students in solving function problems. Furthermore, this research can be developed on how to generalize the relationship between quantity variations in the form of composition functions.

LIMITATION AND FURTHER STUDY

This study only discusses the students' correct answers, therefore the analysis of students' works in solving mathematics problems needs to be conducted further.

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AUTHOR'S CONTRIBUTION

The author is a doctoral student who contributes to writing proposals, conducting research, and reporting the results of the research. Co-authors are professors and senior lecturers who provide input and research ideas.

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