



TEACHERS' COMPETENCE IN THE ACQUISITION AND USE OF THE MATHEMATICAL LANGUAGE IN INDONESIA

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Article History: Received on 24th July 2020, Revised on 23rd August 2020, Published on 4th October 2020

Abstract

The purpose of the study: This paper aimed at describing the teachers' competence in the acquisition and use of the language of mathematics at public and private senior high schools all over Malang City, Malang Regency, and Batu City, Indonesia.

Methodology: Qualitative descriptive research design was employed by means of content analysis on the results of the interview and documents of mathematic instructions designed by 30 teachers who also served as the research subjects, spiral analysis on the qualitative data by Creswell was applied to analyze the data, with the support of NVivo 12 plus software.

Main Findings: This research has revealed that the teachers have acquired and used the language of mathematics, with some errors and fallacies in the use of mathematics vocabularies, grammar, and the explanation of mathematical symbols. The teachers' competence in the acquisition and use of the language of mathematics was influenced by the knowledge about mathematical concepts, presentations, and instructions.

Applications of this study: The findings of this study are expected to add new knowledge about the competency of the mathematics language of the teacher so that it can be used as a reference for colleges producing teacher candidates in developing the curriculum. Theoretically, these results can be used as a reference and contribute to assessing the mathematical language competence of teachers.

Novelty/Originality of this study: This current research, is based on their educational qualifications, and to identify factors that influence the teachers' competence in the acquisition and use of the language of mathematics which is less explored by the earlier researcher.

Keywords: *Teachers' Competence, Language of Mathematics, Vocabulary, Symbols, Instructional Materials.*

INTRODUCTION

Instructions that are focused on the language of mathematics are potentially challenging due to the abstractness of objects under discussion, with the consequence of the difficulty in demystifying something behind the objects (Mulwa, 2015; Schleppegrell, 2007). Therefore, teachers are required to possess eligible competence in the acquisition and use of the language of mathematics. This is because teachers' knowledge is highly influential upon their students' achievement (Murtafiah & Lukitasari, 2019). Teachers who are well competent in the language of mathematics must be effective in presenting the materials to their students, mainly about how to use mathematical expressions, symbols, and multi-representations (Accurso, Gebhard, Purington, 2017).

Teachers play an essential role during instructions since their performance will affect their students' achievement and involvement within the instructions (Ent et al., 2017; Wilkinson, 2018). While actualizing the goals of mathematics instructions, teachers are to use more structured language, encompassing technical vocabularies, and symbols which students really understand (Jupri & Drijvers, 2016; Mulwa, 2015). It is since mathematics, in its nature, is procedural, with the emphasis on specific techniques that involve numerical, symbolic, and equation elements. Patterns and relationships amongst concepts made of symbols and equations remain the ultimate problem which students need to comprehensively understand. For that reason, logics and the language of mathematics are essential to understand (Hajer & Norén, 2017; Prediger, 2019).

Practically, teachers are still perplexed in translating mathematical concepts into proper the language of mathematics (Al-sehli & Maroof, 2020). Despite their mastery of the concepts, they remain confused to define them. They also feel hard to state mathematical problems into symbols or sentences (Lee et al., 2019; Leiss et al., 2019). This fact remains problematic during mathematics instructions since, in essence, teachers are required to provide students with mathematical meanings through several competencies, such as reading, writing, and discussing any mathematical concepts (Accurso, Gebhard, Purington, 2017). Besides, the challenges for mathematics teachers do not only lie on the complexity of the mathematical concepts and some requirements needed, but also on the sophistication of the semiotic system that exists in mathematics (Lane et al., 2019). Therefore, preparing teachers' competence is of great urgency.

Basically, professional and pedagogical competencies that are related to the mastery of the content of material and ways of teaching it to remain the essential aspect for teachers (Murtafiah & Lukitasari, 2019). Teachers are to be equipped with a skill of explaining connections from one particular notation to other notations, or from one specific procedure to other procedures, which is considered one of the indicators of professional competence in addition to using unambiguous

language to demystify reasons behind the mathematical procedures ([Ilany & Hassidov, 2018](#)). By means of mathematics vocabularies as well as definitions, students explore related concepts and learn the meanings ([Barnes & Stephens, 2019](#); [Ilany & Hassidov, 2018](#)). Explaining the language of mathematics to students helps teachers vividly identify what complicates and eases their students to simplify their understanding of the language of mathematics more meaningfully ([Leiss et al., 2019](#)).

Previous studies that are relevant to this current investigation are those claiming that mathematics is hard to understand because of confounding terms and symbols ([Hajer & Norén, 2017](#); [Jupri & Drijvers, 2016](#); [Planas, 2018](#); [Wilkinson, 2018](#)). For that reason, instructions regarding the language of mathematics will elevate the effectiveness of mathematics instructions. It is also indicated that studying mathematical terms is pivotal ([Burke, 2017](#); [Lane et al., 2019](#)). In mathematics instructions, students find it problematic to translate mathematical terms into particular models based on any given situations ([Jupri & Drijvers, 2016](#)).

Many previous studies had indicated that mathematics teachers, at junior and senior high school levels, were equipped with satisfactory competence in understanding the language of mathematics ([Accurso, Gebhard, Purington, 2017](#); [Strutchens, 2018](#)). However, not all teachers with that excellent understanding could explain and deliver the materials precisely using the proper and accurate language of mathematics ([Hajer & Norén, 2017](#); [Wilkinson, 2018](#)). Besides, many of the previous studies also demonstrated that several mathematics teachers had not been skilful enough in explaining and translating mathematical symbols and graphs accurately ([Burke, 2017](#); [Ent et al., 2017](#); [Ilany & Hassidov, 2018](#)). Considerably a few previous studies explored and investigated teachers' competence in the acquisition and use of the language of mathematics based on the standards of educational qualifications. For that reason, this current research is intended to explore how the teachers' competence in the acquisition and use of the language of mathematics is, based on their educational qualifications, and to identify factors that influence the teachers' competence in the acquisition and use of the language of mathematics.

LITERATURE REVIEW

Definition and concept of the language of mathematics

The language of mathematics is defined as a system used by mathematicians to communicate mathematical notions amongst their groups. The language contains natural codes with the use of unique technical terms and grammatical convention to deal with mathematical discourses, with the addition of very special symbolic notations for mathematical formulas ([Ilany & Hassidov, 2018](#)). Typically, regarding the language of mathematics, basic competence in using mathematics dictions, explaining, justifying, and communicating in the sense of mathematics is of the necessity for holistic development on mathematical competencies ([Genc & Erbas, 2019](#); [Lee et al., 2019](#); [Mulwa, 2015](#); [Sibel & Burcu, 2016](#); [Wilkinson, 2019](#)).

The language of mathematics consists of semiotic and grammatical or syntactic features ([Schlepppegrell, 2007](#)). Semiotics is a study of signs that can be in the forms of words, figures, sound, and body or object movements ([Planas & Schütte, 2018](#)). Meanwhile, the syntactic feature is concerned with the relationship between one and other words or elements as a unity. One single syntactic pattern comprises words, phrases, clauses, sentences, and discourses ([Wilkinson, 2018](#)). Semiotic features, moreover, include notations made of mathematical symbols and graphs or visual looks. On the one hand, grammatical features encompass technical vocabularies and implicit logical connections ([Schlepppegrell, 2007](#)). A syntactical unit observed in this current research is focused on occupied vocabularies and sentences.

Syntactical features in the language of mathematics involve a list of symbols, the configuration of rules in producing language patterns, axioms, deductive systems, and theorems. Elements in the language of mathematics include symbols, concepts, definitions, and theorems ([Genc & Erbas, 2019](#); [Rahman et al., 2019](#)). Mathematical terms and symbols are defined clearly. Each of the statements expressed in the language of mathematics only applies to a single meaning, unambiguous. In addition, every mathematical pattern has one single structure that is defined through operational rules ([Ilany & Hassidov, 2018](#)). The graph is one of the examples of visual representations in mathematics. Some other visual representatives can be used to understand mathematical concepts ([Jupri & Drijvers, 2016](#)). In mathematics, the selection of visual representatives depends on what to express ([Accurso, Gebhard, Purington, 2017](#); [Weinberg, 2019](#); [Wilkinson, 2019](#)). For exemplify, visual representatives in mathematics, there are two-dimensional figures and solid geometry.

Many higher-order sentences in mathematics are made of sophisticated grammar that is easy to understand if one knows some basic mathematical terms and vocabularies ([Burke, 2017](#); [Mulwa, 2015](#); [Schlepppegrell, 2007](#)). Satisfactory competence in using the language of mathematics strongly demands basic knowledge of the vocabulary used, flexibility, and skillfulness in understanding numerical, symbolic, and syntactical features ([Wilkinson, 2018](#)). In reality, students still find it problematic to understand the features, such as in formulating definitions based on the relationship between mathematical terms and their entailing concepts ([Mulwa, 2015](#)). [Jupri & Drijvers \(2016\)](#) have formulated mathematical models and transformed given problems into mathematical equations. Students get difficult in defining which one of the operational procedures is the best for use in search of the most effective solution due to their inability to structure the most appropriate mathematical model ([Rahman et al., 2019](#); [Sibel & Burcu, 2016](#)). There is a factor deemed to cause

students to feel puzzled in understanding the language of mathematics. The complex patterns make students unfamiliar with dictions they are exposed to ([Jupri & Drijvers, 2016](#)).

The teachers' competence in understanding the language of mathematics

Mathematics teachers are challenged when preparing and presenting materials to teach their students, which requires them to satisfactorily and accurately possess the basic skill of the language of mathematics ([Aksu & Kul, 2019](#); [Barnes & Stephens, 2019](#)). The teachers are to be competent in the acquisition of concepts and content knowledge of mathematics, pedagogical knowledge, and curriculum knowledge ([Weinberg, 2019](#)). Those who possess the three competencies will demonstrate their acquisition and use of the language of mathematics satisfactorily ([Planas & Schütte, 2018](#); [Wilkinson, 2018](#)). Therefore, the foremost requirement to be a professional teacher of mathematics is to possess knowledge of the language of mathematics, skills in presenting materials with the use of the language of mathematics, and competence in designing materials by means of the accurate use of the language of mathematics ([Cueto et al., 2017](#); [Genc & Erbas, 2019](#)).

[Wilkinson \(2019\)](#) specifically formulates the essential aspect to be understood by mathematics teachers, in which they need to understand the basic language of mathematics, or referred to as 'mathematics register'. Mathematics register refers to a specific definition of the basic language of mathematics that includes the understanding of the grammatical rules, vocabularies, and symbols, such as graphs and figures ([Wilkinson, 2018](#)). In addition, [Wilkinson \(2019\)](#) emphasizes that mathematics teachers need to alternate their point of view that mathematics education is a study full of language, symbols, and vocabularies; each of which underlies hidden meanings that are different from that of in common language. During mathematics instructions, teachers are to make use of the language of mathematics properly and accurately ([Golding, 2017](#); [Hill et al., 2008](#); [Purpura & Reid, 2016](#)). Accordingly, mathematics teachers are supposed to be careful and considerate in understanding the grammatical rules, using proper diction, and translating symbols accurately and correctly ([Planas, 2018](#); [Prediger, 2019](#)).

Another competence mathematics teachers should acquire to understand the mathematics curriculum in which teachers are demanded to design materials and goals of mathematics instruction with the use of the comprehensible language of mathematics so that students can easily understand ([Cueto et al., 2017](#); [Handal & Herrington, 2003](#); [Murtafiah & Lukitasari, 2019](#); [Umar, 2012](#)). Clear and comprehensible materials and goals of instruction for students can affect the accomplishment of targets of instruction, to name the formation of intelligence, knowledge, and skills in mathematics, including to use and translate instructional materials into the language of mathematics accurately ([Jupri & Drijvers, 2016](#)). Mathematics teachers are considered professional only if they are found competent in designing materials and goals of instruction that appear to activate their students to effectively understand and acquire the language of mathematics ([Hiebert et al., 2017](#); [León Gómez & Beyer Kessler, 2017](#); [Ortega et al., 2018](#)).

The abovementioned elaboration has strongly remarked that an analysis of the teachers' competence in the acquisition and use of the language of mathematics refers to the analysis of the teachers' competence in understanding the grammatical rules, using proper vocabularies, and translating mathematical symbols, which is highly dependent on the content knowledge, the presentation of materials and innovation of instruction, and the design of instructional materials based on the convention of the language of mathematics ([Planas & Schütte, 2018](#)). This current research highlights the teachers' competence in the acquisition and use of the language of mathematics and the teachers' familiarity with the concepts, presentation, and material designing by means of the accurate language of mathematics.

METHODOLOGY

This research met the sense of qualitative descriptive design, which described the teachers' competence in the acquisition and use of the language of mathematics employing content analysis approach. This current research was also completed by data triangulation, comprising interviews and documents of instructional materials designed by mathematics teachers who were recruited as the research subjects. The subjects, further, were selected by using purposive sampling technique, picking up 30 mathematics teachers classified into three groups based on their educational qualifications, namely Bachelor's degree, Master's degree, and Doctorate's degree, teaching at public and private senior high schools in Malang City, Malang Regency, and Batu City, Indonesia. The purpose of using purposive sampling in this study is to obtain qualitative information and data in the form of interviews, documents, and observations that can answer research questions comprehensively. The purposive sampling approach in qualitative research is implemented through the determination of research informants who can represent and describe the problems being studied. Therefore, this study determined 30 mathematics teachers as research informants classified based on educational qualifications, type of school, and research location. Based on educational qualifications, the informants of this study consisted of three groups, namely mathematics teachers with bachelor degrees (10 people), masters (10 people), and doctorates (10 people). Meanwhile, based on the type of school, the research informants consisted of two groups, namely state schools and private schools which were selected proportionally in three locations, namely Malang City, Malang Regency, and Batu City.

The consideration of using educational qualifications to recruit the subjects was intended to investigate the difference between the teachers' competences based on their levels of academic qualifications. Meanwhile, the consideration of selecting public and private schools was aimed at collecting data from mathematics teachers whose instructional

materials were designed utilizing standardized the language of mathematics for the high school level. Therefore, the data could be used to describe the teachers' competence in the acquisition and use of the language of mathematics comprehensively.

Table 1: The subjects of the research on the teachers' competence in the acquisition and use of the language of mathematics

Names	Education Degrees	Sex	Type of School	Location
Informant 1	Bachelor's degree	Male	Public	Malang City
Informant 2	Master's degree	Female	Private	Malang City
Informant 3	Master's degree	Male	Public	Malang City
Informant 4	Bachelor's degree	Female	Private	Malang City
Informant 5	Doctorate's degree	Male	Public	Malang City
Informant 6	Bachelor's degree	Female	Public	Malang City
Informant 7	Master's degree	Male	Public	Malang City
Informant 8	Doctorate's degree	Female	Private	Malang City
Informant 9	Bachelor's degree	Male	Public	Malang City
Informant 10	Master's degree	Female	Private	Malang Regency
Informant 11	Doctorate's degree	Male	Public	Malang Regency
Informant 12	Bachelor's degree	Male	Private	Malang Regency
Informant 13	Master's degree	Female	Public	Malang Regency
Informant 14	Doctorate's degree	Male	Private	Malang Regency
Informant 15	Bachelor's degree	Female	Public	Malang Regency
Informant 16	Master's degree	Male	Private	Malang Regency
Informant 17	Doctorate's degree	Female	Public	Malang Regency
Informant 18	Bachelor's degree	Male	Private	Malang Regency
Informant 19	Master's degree	Female	Public	Malang Regency
Informant 20	Doctorate's degree	Male	Private	Batu City
Informant 21	Bachelor's degree	Female	Public	Batu City
Informant 22	Master's degree	Male	Private	Batu City
Informant 23	Bachelor's degree	Female	Public	Batu City
Informant 24	Doctorate's degree	Female	Private	Batu City
Informant 25	Master's degree	Male	Public	Batu City
Informant 26	Doctorate's degree	Female	Private	Batu City
Informant 27	Bachelor's degree	Male	Public	Batu City
Informant 28	Master's degree	Female	Private	Batu City
Informant 29	Doctorate's degree	Male	Public	Batu City
Informant 30	Bachelor's degree	Female	Private	Batu City

The spiral qualitative analysis technique designed by [Creswell \(2017\)](#) was used for data analysis. [Creswell \(2017\)](#), in addition, designed the technique into five main phases, namely (1) organizing data, (2) reading and making out the memo, (3) describing, classifying, and translating data into codes and themes, (4) interpreting data, and (5) presenting and visualizing data. Further, Qualitative Analysis Data Software (QADS) was employed to analyze the data from the interview and documents of instructional materials that could obtain quality data in order to describe the teachers' competence in the acquisition and use of the language of mathematics. QADS Nvivo 12 plus was used to ease the procedures of data collection, management, and analysis to provide credible data and valid findings to be presented further ([Wallmeier et al., 2019](#)).

The essential phase in data analysis was text coding on the script of the interview and documents of instructional materials designed by the teachers. The text coding was the categorization of the script of interview and documents of mathematics instructional materials into topics/themes on the basis of concepts related to the teachers' competence in the acquisition and use of the language of mathematics ([Krippendorff, 2004](#)). There were also some steps of data coding, such as (1) making nodes, in which the names of the nodes were adjusted to the concept of the teachers' competence in the acquisition and use of the language of mathematics; therefore, the nodes could be used to answer the questions; and (2) data coding that consisted of reading through the script of interview and documents of mathematics instructional materials, understanding the texts and inserting the texts into the nodes for the need of analysis using available features in Nvivo 12 plus software ([Maher et al., 2018](#)).

A crosstab feature in Nvivo 12 plus software was used for data analysis on the comparison of the teachers' competence from different academic qualifications, such as Bachelor's degree, Master's degree, and Doctorate's degree in the acquisition and use of the language of mathematics. At this point, the use of the crosstab feature in Nvivo 12 plus software was aimed at answering the question of, "how is the teachers' competence in the acquisition and use of the language of mathematics (grammar, vocabulary, and symbol)?" Further, data analysis by means of chart feature was

meant to answer the question of, "what are the factors that influence the teachers' competence in the acquisition and use of the language of mathematics?"

RESULTS

The teachers' competence in the acquisition and use of the grammar of mathematics

Regarding the results of coding on the documents and interview by means of Nvivo 12 plus software, it was indicated that the teachers with Bachelor's degree qualification had shown better competence in the acquisition and use of the grammar of mathematics than those with Master's and Doctorate's degree qualifications. Figure 1 shows that the teachers with Bachelor's degree qualifications were equipped with competence in the acquisition and use of the grammar of mathematics with the portion of 208.48%; while those with Master's and Doctorate's degree qualifications performed consecutively in the portion of 171.18% and 72.83%. The data finding confirmed that the teachers with Bachelor's degree qualifications were more competent in the acquisition and use of the grammar of mathematics than their counterparts with Master's and Doctorate's degree qualifications.

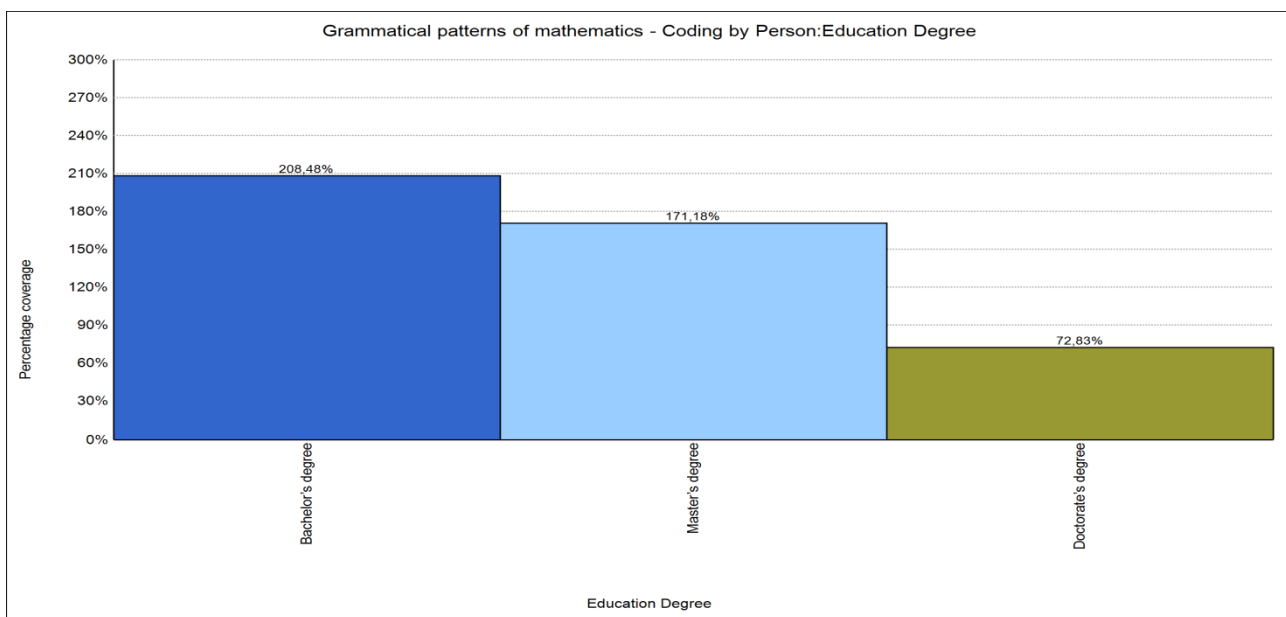


Figure 1: The teachers' competence in the acquisition and use of the grammar of mathematics

Figure 1 affirms that the teachers with a Bachelor's degree qualification performed a satisfactory understanding of the use of the grammar of mathematics, which means that they were able to formulate the grammar of mathematics accurately. They were competent in giving instructions and directions to their students by means of proper and accurate compositions based on the grammar of mathematics. Therefore, students feel certain in understanding the materials of mathematics instruction. In addition, the teachers with Master's and Doctorate's degree qualification also showed a good understanding of the grammar of mathematics, but less competent in presenting the grammar in the forms of accurate compositions based on the conventions of the language of mathematics.

The teachers' competence in the acquisition and use of mathematics vocabularies

The teachers' competence in the acquisition and use of the grammar of mathematics as elaborated above included the competence in the use of mathematics vocabularies, encompassing the accuracy on the use of proper diction in defining mathematical symbols. In this case, Figure 2 indicates that the teachers with Bachelor's degree qualification (220.89%) were more competent than those with Master's degree qualification (196.82%) and Doctorate's degree qualification (54.81%) in the use of vocabularies related to the rational relationship with symbols. The teachers with Bachelor's degree qualification compiled documents of mathematics instruction by means of proper vocabularies related to symbols, which means that the written vocabularies were rationally connected with the represented symbols.

The teachers with a Bachelor's degree qualification were slightly different from those with a Master's degree qualification in using vocabularies rationally related to symbols. In spite of the disparity regarding the difference, both groups of the teachers showed a typical degree of competence. In this case, the teachers with Master's degree qualifications were also competent in using vocabularies rationally connected with the mathematical symbols; yet they lacked carefulness and considerateness in using proper vocabularies, which resulted in an inaccurate relationship between the vocabularies and mathematical symbols. Meanwhile, the teachers with Doctorate's degree qualification were the least competent amongst the whole groups in using vocabularies rationally related to the mathematical symbols.

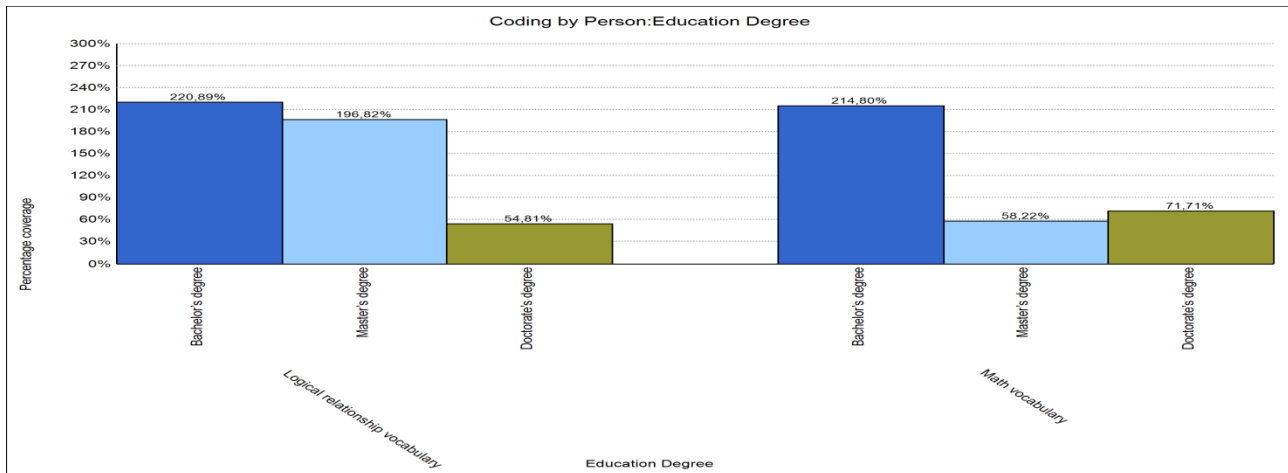


Figure 2: The teachers' competence in the acquisition and use of mathematics vocabularies

Figure 2 indicates the teachers' competence in using correct vocabularies to define mathematical symbols when delivering the instructional materials. The results of document analysis of the instructional materials by means of Nvivo 12 plus software indicated that the teachers with Bachelor's degree qualification were more competent than those with Master's and Doctorate's degree in using accurate vocabularies. This finding clarified that the teachers with Bachelor's degree qualification were competent in the acquisition and use of the grammar of mathematics, including the use of vocabularies for translating and defining mathematical symbols. Meanwhile, the teachers with Master's degree and Doctorate's degree qualifications were slightly different in terms of competence; the latter showed up better competence in the use of correct and accurate vocabularies.

The teachers' competence in the acquisition and use of mathematical symbols

The teachers' competence in the acquisition and use of the language of mathematics was not only dependent on the accurate use of vocabulary, but also on the use of semiotics of mathematics accurately. Figure 3 indicates that the teachers with Bachelor's degree qualifications were better incompetent than those with Master's and Doctorate's degree qualification, primarily in the use of symbols (signs and formulas) of mathematics accurately, consecutively with the scores of 197.40%, 177.38%, and 70.12%. The finding affirmed that the teachers with Bachelor's degree qualifications showed a better understanding of the grammar of mathematics. Meanwhile, the teachers with Master's degree qualification had a better understanding than those with Doctorate's degree qualification.

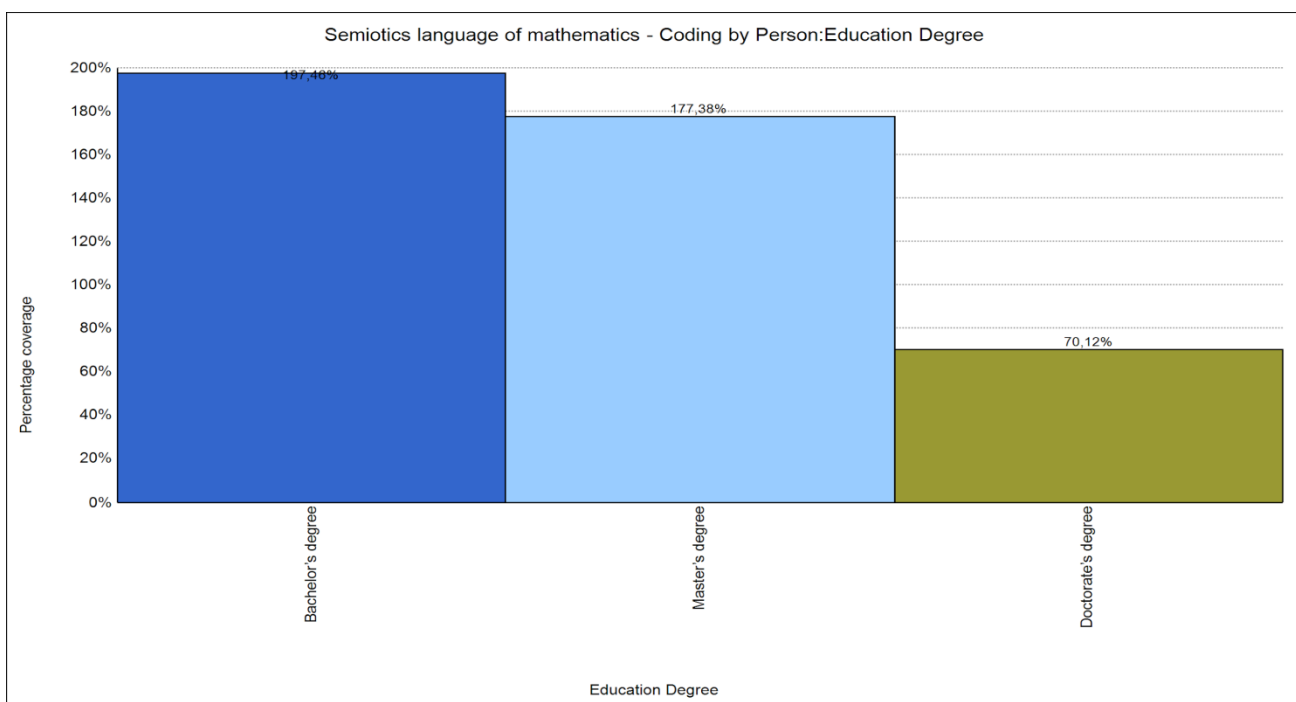


Figure 3: The teachers' competence in the acquisition and use of mathematical symbols

Despite the fact that the Bachelor's degree graduates were better than the Master's and Doctorate's degree graduates in the acquisition and use of the language of mathematics, this does not necessarily mean that the latter were incompetent

in understanding the grammatical concepts of mathematics satisfactorily. The inability shown by the teachers with Master's and Doctorate's degree qualifications in the acquisition of the grammar of mathematics was influenced by the fallacies and carelessness in formulating and designing materials written on the documents of instruction as exemplified in the following table.

Table 2: The fallacies in the use of the language of mathematics by Master's and Doctorate's degree graduates

No.	Inappropriate words/phrases/sentences	Corrections for the inappropriate words/phrases/sentences	Explanation
1.	$h = -1$ $\rightarrow (-1)^3 - 11(-1)^2$ $+ 30(-1) - 8$ $= -28$	$h = -1, \text{ therefore}$ $f(x) = (-1)^3 - 11(-1)^2$ $+ 30(-1) - 8$ $= -28$	The inaccurate implicit logical relationship appeared due to the exclusion of the notation of $f(x)$ by the teachers, with the hope that students understand the origin of the substitution procedure.
2.	$4\left(-\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^2 - k\left(\frac{3}{2}\right)$ $+ 2\left(\frac{1}{4}\right)$ $-\frac{27}{2} - \frac{9}{4} - \frac{3}{2}k + \frac{9}{4}$ $-\frac{27}{2} - \frac{3}{2}k = 0$ $-\frac{3}{2}k = \frac{27}{2}$ $k = 9$	$4\left(-\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^2 - k\left(\frac{3}{2}\right) + 2\left(\frac{1}{4}\right)$ $= 0$ $-\frac{27}{2} - \frac{9}{4} - \frac{3}{2}k + \frac{9}{4} = 0$ $-\frac{27}{2} - \frac{3}{2}k = 0$ $-\frac{3}{2}k = \frac{27}{2}$ $k = 9$	The unsystematic use of the concepts due to the absence of $f(x) = 0$ requirement at the beginning of answering the question
3.	$(x^4 + 5x^3) \div (x + 3)$, or x^3	$(x^4 + 5x^3) \div (x + 3)$ results in x^3 as the first terms of the quotient	The diction might raise confusion in an attempt to understand the concept.
4.	Polynomial division of $g(y)$ by $(ax = b)$	Polynomial division of $g(y)$ by $(ax + b)$	The fallacy in the formulation
5.	Sometimes, the teachers notated the polynomial operation with $F(x)$, and somehow with $f(x)$	Consistently and merely using the $f(x)$ notation	Inconsistence of the use of notation

The above table has shown that there were some fallacies in the use of the language of mathematics by the teachers with Master's and Doctorate's degree qualifications. The understanding of the language of mathematics was considered lower than those of Bachelor's degree graduates. Some fallacies committed in the use of the language of mathematics by the teachers with Master's and Doctorate's degree qualifications included: 1) inaccurate implicit logical relationship; 2) fallacy in distinguishing concepts; 3) unsystematic use of concept; 4) use of confusing vocabularies; 4) fallacy in the formulation of symbols; 5) fallacy in the use of symbols, and 6) use of inconsistent notation.

The acquisition factors of the language of mathematics

With reference to the cluster analysis using Nvivo 12 plus software on the teachers' competence in the acquisition and use of the language of mathematics, the teachers with Bachelor's degree qualification (191.61%) were shown pedagogically more competent than those with Master's (124.82%) and Doctorate's (80.21%) degree qualifications as shown in Figure 4. The pedagogical competence in the acquisition and use of the language of mathematics was related to the teachers' competence in presenting and explaining the materials, including grammatical rules, vocabulary use, and translation of mathematical symbols accurately to their students. Therefore, the students could eventually understand the instructional materials accurately.

In spite of the fact that the Master's and Doctorate's degree graduates were less competent than the Bachelor's degree graduates, the formers were equipped with more satisfactory content knowledge, with the Master's degree graduates signifying 145.23%, the Doctorate's degree graduates accounting for 122.17%, and the Bachelor's degree graduates showing 77.87% in the acquisition and use of the grammar of mathematics. Moreover, the content knowledge was related to the teachers' understanding of the contents of the materials, such as the grammar of mathematics, vocabularies, and mathematical symbols. The teachers with Master's and Doctorate's degree qualifications were better in understanding and explaining using the proper language of mathematics, shown by their competence in translating the meanings of the mathematical symbols using accurate vocabularies and grammar. Meanwhile, the teachers with a Bachelor's degree qualification were less competent in terms of content knowledge than those with Master's and Doctorate's degree qualifications.

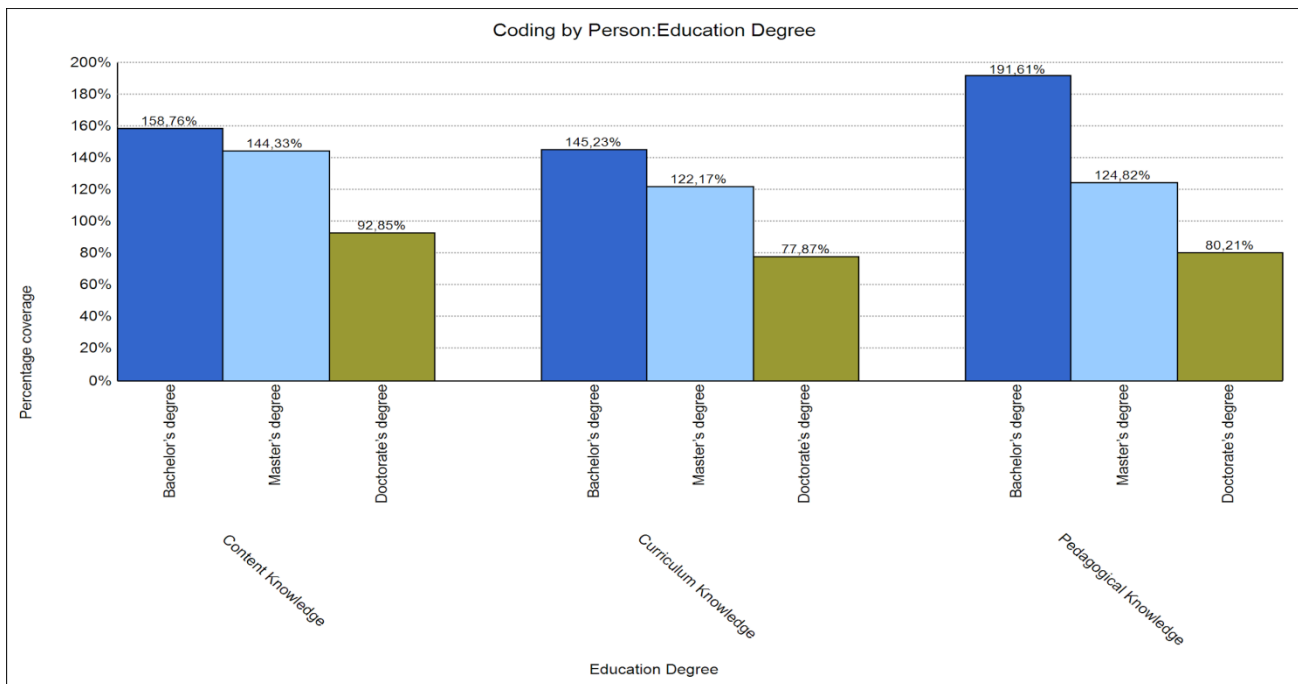


Figure 4: The factors of the teachers' competence in the acquisition and use of the language of mathematics.

DISCUSSION

In general, most of the teachers showed a satisfactory understanding of the materials. However, this does not guarantee that the teachers could explain their ineffective ways to their students (Afifah et al., 2019). Despite their acquisition of the concept, the teachers were still found confused in explaining mathematical concepts by means of the accurate language of mathematics, which the students could easily understand (Leiss et al., 2019). This was indicated by some fallacies in their language of mathematics. Some of the fallacies committed by the mathematics teachers were referred to as aspects of semiotics and grammatical patterns (Wilkinson, 2018). The fallacies in semiotics included the fallacy in using symbols, the inconsistency in using symbols, and the fallacy in formulating the operational symbols (Makamure & Jita, 2019; Schleppegrell, 2007). Meanwhile, the fallacies in grammatical patterns encompassed the implicit logical relationship that was inaccurate and unsystematic in the application of the concept and the use of ambiguous vocabularies (Barnes & Stephens, 2019; Ilany & Hassidov, 2018).

Sometimes, the teachers were inconsistent in using symbols. In addition, the implicit logical relationship of the sentence was not prepared effectively. Both were the fallacies committed by the teachers in understanding the concept that was not supposed to happen (Wilkinson, 2018). There were some concepts of mathematics that the students find them difficult to understand (Jupri & Drijvers, 2016). In this case, the teachers should consider strategies and innovations of material presented by the use of the proper language of mathematics that is comprehensible and understandable for the students (Al-sehli & Maroof, 2020; Planas, 2018). The teachers should be careful and considerate in making use of mother language in an attempt to explain the materials. At this point, the teachers are to avoid the use of any language which was irrelevant to mathematics (Emre-Akdoğan & Yazgan-Sağ, 2018; Ent et al., 2017; Murtafiah & Lukitasari, 2019).

Furthermore, the teachers from all groups of various academic qualifications were different in their levels of the acquisition and use of the language of mathematics. The teachers with Bachelor's degree qualification were shown better in the acquisition and use of the language of mathematics than those with Master's and Doctorate's degree qualifications. The former was equipped with the competence of using proper vocabularies and explaining as well as translating mathematical symbols accurately. This sort of competence was inseparable from their intensity of getting involved in teaching and learning practices (Rahman et al., 2019; Strutchens, 2018). This finding has confirmed that the teachers' experiences influenced the level of their competence in explaining using the proper language of mathematics accurately (Burke, 2017; Cueto et al., 2017).

Despite their not possessing the competence in the acquisition and use of the language of mathematics compared to the teachers with Bachelor's degree qualification, it does not necessarily mean that the teachers with Master's and Doctorate's degree qualifications were incompetent in understanding the grammar and symbols of mathematics (Sibel & Burcu, 2016). Nonetheless, they were less competent in the formulation, application, and presentation of the grammar of mathematics when delivering the materials (Strutchens, 2018). This finding has postulated that the teachers who were equipped with a good understanding on the grammar of mathematics did not necessarily show satisfactory performance in the presentation and explanation of the information they imparted (Wilkinson, 2018). This has clarified that the

teachers, at least, have to be well knowledgeable about content knowledge, pedagogical knowledge, and curriculum knowledge (Ent et al., 2017; Norton, 2018; Wilkinson, 2019).

Parallel with the findings above, the teachers with Master's and Doctorate's degree qualifications possessed better conceptual knowledge of the grammar of mathematics than those with a Bachelor's degree qualification. This indicated that the formers were competent in the acquisition of conceptual knowledge about the language of mathematics, including dictions and the translation of meanings and definitions of mathematical symbols. They acquired that competence during their study in Master's and Doctorate's degree programs, primarily focusing on the reinforcement and improvement of the conceptual understanding of the materials for mathematics instructions (Schlepppegrell, 2007; Strutchens, 2018). In addition, the teachers with Master's and Doctorate's degree qualifications were also found satisfactory in understanding the curriculum of mathematics, with the emphasis on the reinforcement of the understanding of the curriculum policies of mathematics (Burke, 2017; Strutchens, 2018).

The teachers with a Bachelor's degree qualification were competent in the presentation and explanation of the materials about the grammar of mathematics. This finding has affirmed that they were competent in presenting the materials using the proper language of mathematics, directly or indirectly taught to their students (Burke, 2017; Ent et al., 2017). They were effective in formulating relevant vocabularies accurately in the process of translating the meanings and definitions of symbols, signs, and formulas in mathematics, which were easily understood by their students (Barnes & Stephens, 2019; Ilany & Hassidov, 2018). In addition, the teachers with Bachelor's degree qualification were able to make use of symbols and explain them accurately so that the students could easily understand the materials, which also affected their students' involvement and participation in the instructional activities (Imre, 2017). The competence in the presentation and explanation of instructional materials was inseparable from the teachers' experiences and active participatory in a series of instructional activities (Handal & Herrington, 2003; Makamure & Jita, 2019; Strutchens, 2018).

CONCLUSION

This research has indicated that the teachers could understand a list of semiotic features correctly, such as a sigma symbol, as well as explaining and translating the symbols and signs of the language of mathematics. It was also necessary that the teachers could create some grammatical patterns in the forms of vocabularies and logical relationship. Notwithstanding, there were some fallacies in understanding the language of mathematics. It happened to the semiotic features, such as misuse of symbols, inconsistent use of symbols, and misuse of the formulation. Meanwhile, the fallacies regarding grammatical features were found on the application of inaccurate and unsystematic implicit relationship to the concepts and the use of confusing vocabularies.

The teachers, whatever their academic qualifications were, showed the difference in the levels of competence and knowledge regarding the acquisition and use of the language of mathematics, influenced by the different competence they showed in the acquisition of the concepts, presentation, and curriculum of mathematics. The teachers with Bachelor's degree qualification were found more competent in presenting the materials than those with Master's and Doctorate's degree qualifications. Meanwhile, the latter was shown better in the acquisition of the concepts of the language of mathematics, but less competent in presenting and explaining the materials using the proper language of mathematics. Therefore, this study emphasizes that overall mathematics teachers need to improve their ability to understand the concepts and symbols of mathematics learning material because mastery of learning material is the key to the success of teachers in increasing students' abilities correctly and correctly. In addition, teachers are also required to understand the learning curriculum properly and correctly because the curriculum is a learning guide for teachers in achieving mathematics learning goals and targets. Another aspect that must be mastered by mathematics teachers is learning strategies that support students in understanding learning material easily and pleasantly.

LIMITATION AND STUDY FORWARD

The current study explored and investigated teachers' competence in the acquisition and use of the language of mathematics based on their educational qualifications, and to identify factors that influence the teachers' competence in the acquisition and use of the language of mathematics. Those who are able to acquire knowledge are considered professional teachers in performing their role as the real mathematic teachers. For further researches, it is recommended to elaborate the quality and professionalism of mathematic teachers, focusing on the discussion of the quality and competence of mathematic teachers in acquiring, presenting, and explaining instructional materials to their students in high school levels by means of qualitative and quantitative approaches assisted by a supportive data analysis software.

AUTHORS CONTRIBUTION

Baiduri has acted as a planner, executor, analyzing research results, writing scripts, and providing key conceptual ideas. Dwi Priyo Utomo developed theories, analyzed data, and wrote scripts with the input of all authors. All authors provide critical feedback and help in the analysis of the manuscript.

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