

MODELLING THE CONDITIONAL CO-MOVEMENTS OF PAKISTAN AND INTERNATIONAL STOCK MARKETS

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Article History: Received on 17th April 2021, Revised on 2nd May 2021, Published on 10th May 2021

Abstract

Purpose of the study: This study assesses and evaluates the conditional co-movements and dynamic conditional correlation of the Pakistan Stock Exchange (PSX) with other Stock Market.

Methodology: DCC-GARCH model has been applied due to its feasibility to model the covariance as a function of correlation and variance together.

Main findings: The findings of the research suggest that the Pakistani Stock Exchange (PSX) is highly volatile compared to two other selected stock markets. In-sample fitting, the study has selected the DCC-GARCH (1, 1) model based on information criterion, conversely, the criterion used for out-of-sample forecast evaluation such as MSFE, RMSFE, MAPE, selected the DCC (2,1)-GARCH (1,1).

Application of the study: This study is very useful for the Pakistan stock market and other international selected stocks markets until and unless the government of Pakistan and other governments will devise new policies which may open new opportunities to investors.

Novelty/ Originality of the study: This study has a great potential in the Pakistani stock market to offer investors to several foreign and domestic investors, allowing them to hold Pakistan as well as foreign and local stocks all major benefits.

Keywords: *Modelling, Conditional Co-movements, Pakistan Stock Exchange, International Stock Exchange.*

INTRODUCTION

Introduction: Correlation measures the relationship between variables. It also plays an essential role in finance and economic related problems such as pricing the financial products and hedging strategies (Ehrhardt *et al.*, 2016). Usually, the correlation coefficient is assumed to be constant in many models while in the financial quantity real market, it is strongly nonlinear (Teng *et al.*, 2016). Similarly, co-movement defines the correlation or movement among two or more entities (Nasreen *et al.*, 2020). It is descriptive rather than explanatory (Evans and McMillan, 2009). Measuring co-movements between stock markets is a widely debated issue and has turned into an empirical area of investigation. Monthly stock market index returns studies of co-movement have increased in recent years and there is a varying asymmetry of negative and positive co-movements between stock markets (Kizys and Pierdzioch, 2009). Whereas in financial literature stock market, co-movements are not constant (Ahlgren and Antell, 2010; Lal, 2019; Vacha and Barunik, 2012).

Motivation: So far as Pakistan Stock Market is concerned, very few studies have been carried out regarding its co-movement with other stock markets and inadequate literature is available that analyzes the linkages between markets at global and local levels with Pakistan as a base market by using rigorous econometric modelling approach such as the Multivariate-GARCH (MGARCH) (Alvi and Chughtai, 2015; Ghufuran *et al.*, 2016; Iqbal, 2014). Apart from a few studies, the literature provides no detailed testing of specific cross-mean and volatility spillover between Pakistan and international stock markets. Among the several characteristics of the financial markets, one which is well documented in the economic and finance literature is that changes in the volatility (standard deviation) appear in clusters of low and high volatility. The consistency in these low and high periods results in a cluster which is known as volatility clustering, a phenomenon introduced by (Cont, 2019; Engle and Patton, 2001; Herbert *et al.*, 2019; Iyiegbuniwe *et al.*, 2012).

Aim of the study: In the present study, the DCC-MGARCH model is applied for capturing volatility and linkages between Pakistan Stock Market (PSX) with International Stock Markets (US & German). The present research is designed to examine the current state of Pakistan, US, and German stock markets and to explore the dynamic linkages (conditional co-movements) of stock market volatility among Pakistan, US, and German Stock Market Returns.

Research Question: The early literature on stock market co-movement found a method that provides a trade-off between time and frequency aspect data. Specifically, the application of DCC-MGARCH is carried out to study the frequency components of time series without losing the time information (Anjum, 2020; Rangel & Engle, 2012; Uddin, 2013). This study conduct on stock market interactions, which has difficult to see any other modern techniques. Like

DCC-MGARCH is a more powerful method to examine as compare to other time series or frequency-based estimation methods because those are based on parameters and estimation methods (Piljak, 2013). Therefore, the applications of this model in the financial stock market co-movements can provide numerous insights into the changing shapes of stock market developments (Ewing & Seyfried, 2003).

The Dynamic Conditional Correlation Multivariate-GARCH (DCC-MGARCH) approach provides estimates for bivariate and multivariate stock market movements for every time-period t (Engle, 2000; Juyo, 2019; Savva, 2009).

Interest in financial markets and the possibility to forecast their sequence is connected to a growing recognition among economists, financial analysts, and policymakers of the increasing impact of financial variables on the world economy and thus on economic policy in general (Baur, 2019). The purpose of this study is to model and quantify conditional correlations and volatility of returns on different stock markets like the Pakistan Stock Market with the DCC-GARCH model. The present study will be fruitful for investors, economists, and policymakers as well to understand the unpredictable behaviour among the three international stock markets i.e., Pakistan Stock Exchange (PSX).

LITERATURE REVIEW

Any analysis of current correlations among the assets is a vital piece of research to have before investors make an investment decision. As a rule, they favour investments with low risk and high return. According to the basic modern portfolio theory, investing in various types of securities reduces risk. The idea of portfolio diversification was expanded on by Das et al. (2018) who advocated an approach that diversifies the portfolio by including foreign markets with a low correlation (Paramati et al., 2016; Trivedi et al., 2021). A number of international financial markets have been investigated in recent years by researchers. These studies showed that there were a wide range of relationships between foreign stock markets (Nguyen & Lam, 2017). A global approach is being taken to these studies, as they concentrate on countries outside of trade blocs as well as those with trade blocs (Daelemans et al., 2018).

Khan et al. (2021), the literature offers benefits and problems associated with business integration (Paramati et al., 2018). More investors see convergence as less as a source of strength and more as a drawback to the goal and simplification of simplifying markets (Ferreira, 2017; Pizado et al., 2020). The increased market integration is credited with contributing to growth, and the well- with greater economic benefit is considered to be a statement paraphrase to improve overall wealth distribution in literature (Panda & Nanda, 2018; Chevallier et al., 2018). However, if capital inflows aren't adequately distributed, financial uncertainty and contagion may be the result (Das & Manoharan, 2019). An important shortcoming in portfolio diversification that became widely recognised after the GFC [the Global Financial Crisis] as buy-and-and-hold investors started to increasingly use the market as a vehicle for holding investments was that global portfolios had lower diversification and higher volatility (Jiang et al., 2017; Hanif, 2020).

The literature has proven that the following factors have a major influence on international stock market integration (Ismail et al., 2016). Although this may be true, there was far less trade among the countries than between companies or even among industry sectors, which were found to be important as separate categories of trade interdependencies (Hkiri et al., 2018; Lim & Masih, 2017). These reports, which were conducted by Gupta and Elyan and Company (along with another three other researchers) asserted that bilateral trade had no impact on national stock market synchronisation (Sakti et al., 2018; Gkillas et al., 2019). Many countries are now trading partners, which has led to more empirical studies being conducted in the long- the long-term connections of financial markets, but these studies have only looked at a few countries (Khan et al., 2020).

METHODOLOGY

The secondary data used in the present study consist of weekly stock market prices which were collected from the official website of Yahoo Finance (<https://finance.yahoo.com>). The data span from January-2004 to December-2017 which yielded 729 observations. The returns were calculated using the first difference of natural logarithm of prices i.e.

$r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$. These returns were then multiplied by 100 to convert them into percentage returns. These percentage

returns were used for further analysis. The return data yield 728 observations. MATLAB (MATRIX LABORATORY) 2015a version is used for analysing the data set. Since MATLAB is a programming language, so all the analysis was done through programming. Econometrics and Statistics Toolboxes were used for this purpose. The results of the test applied would be justified in this section evaluating their findings and provide an interpretation to each of the findings.

In this section, we will discuss the methodology briefly.

Multivariate GARCH Model

MGARCH models are defined as:

$$r_t = \mu_t + \alpha_t \quad (1)$$

$$\alpha_t = H_t^{1/2} z_t \quad (2)$$

However, in this research work, we do focus on the MGARCH model.

DCC-MGARCH

(DCC)-MGARCH represents the class of models that are employed to model conditional variances and correlations. The basic purpose of this model is that covariance matrix H_t can be broken down into the two other components, the conditional standard deviations, D_t , and a correlation matrix, R_t , and both are then assumed that both are time-varying components. (Engle and Sheppard, 2001).

DCC-Model of Engle & Sheppard: Suppose we have a matrix of returns r_t from n assets with conditional expected value $E(r_t) = \mu_t = 0$ and conditional covariance matrix H_t .

(DCC)-MGARCH model is defined as

| | |
|-----------------------|-----|
| $r_t = \mu_t + a_t$ | (3) |
| $a_t = H_t^{1/2} z_t$ | (4) |
| $H_t = D_t R_t D_t$ | (5) |

The elements in the diagonal matrix D_t are standard deviations from univariate GARCH models.

$$D_t = \begin{bmatrix} \sigma & 0 & 0 & 0 \\ 0 & \sigma & 0 & 0 \\ 0 & 0 & \sigma & 0 \\ 0 & 0 & 0 & \sigma \end{bmatrix}$$

Where

$$h_{it} = \alpha_{i0} + \sum_{q=1}^{Q_i} \alpha_{iq} \alpha_{i,t-q}^2 + \sum_{p=1}^{P_i} \beta_{ip} h_{i,t-p}$$

In general, GARCH model has many orders with different parameters, GARCH (1, 1), is the simplest and adequate for modelling log returns. The description of UGARCH model is not narrow as the SGARCH(p,q) model, this is because it is assumed that any GARCH process follows the gaussian distributed random errors which ensure that it fulfills the appropriate stationarity conditions for the existence of unconditional variances. The matrix of conditional correlation R_t for the standardized disturbances random error ε_t , define as:

$$\varepsilon_t = D_t^{-1} a_t \sim N(0, R_t)$$

Since R_t is a correlation matrix it is symmetric.

$$R_t = \begin{bmatrix} 1 & P_{12,t} & P_{13,t} & \Lambda & P_{1n,t} \\ P_{12,t} & 1 & P_{23,t} & \Lambda & P_{2n,t} \\ P_{13,t} & P_{23,t} & 1 & O & M \\ M & M & O & O & P_{n-1,n,t} \\ P_{1n,t} & P_{2n,t} & \Lambda & P_{n-1,n,t} & 1 \end{bmatrix}$$

The elements of $H_t = D_t R_t D_t$ is:

| | |
|--|-----|
| $[H_t]_{ij} = \sqrt{h_{it} h_{jt}} p_{ij}$ | (6) |
|--|-----|

where $p_{ii} = 1$

As stated in the last section there exist various forms of R_t . There are two requirements when we specify the R_t .

1. Matrix H_t must be positive definite.
2. The correlation between all existing elements must be equal or less than one.

If these requirements are met, the DCC-GARCH model, R_t is decomposed into:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \quad (7)$$

$$Q_t = (1 - a - b) \bar{Q} + a \varepsilon_{t-1} \varepsilon_{t-1}^T + b Q_{t-1} \quad (8)$$

where $\bar{Q} = Cov[\varepsilon_t \varepsilon_t^T] = E[\varepsilon_t \varepsilon_t^T]$ and \bar{Q} is estimated as:

$$\bar{Q} = \frac{1}{T} \sum_{t=1}^T \varepsilon_t \varepsilon_t^T$$

Where a and b are two scalar parameters, and they also exist a diagonal matrix which is denoted by Q_t^*

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11t}} & 0 & \Lambda & 0 \\ 0 & \sqrt{q_{22t}} & 0 & M \\ M & 0 & 0 & 0 \\ 0 & \Lambda & 0 & \sqrt{q_{nn}} \end{bmatrix}$$

to fulfill the second condition $|p_{ij}| = \left| \frac{q_{ijt}}{\sqrt{q_{iit}q_{jjt}}} \right| \leq 1$, Q_t^* rescales all the elements in diagonal of. Moreover, the diagonal Q_t which is the square root of the diagonal matrix Q_t^* has to be positive definite, also there exist some conditions on scalar parameters to ensure H_t to be a positive definite matrix, additionally, the scalars a and b must satisfy:

$$a \geq 0, b \geq 0 \text{ and } a + b < 1$$

for the UGARCH model to ensure positive unconditional variances.

Additionally, Q_0 , which is initializing the value of Q_t , ought to be positive definite to ensures H_t to be positive definite, then the general DCC(M, N)-GARCH model for the correlation structure can be extended as:

$$Q_t = (1 - \sum_{m=1}^M a_m - \sum_{n=1}^N b_n) \bar{Q}_t + \sum_{m=1}^M a_m \partial_{t-1} \partial_{t-1}^T + \sum_{n=1}^N b_n Q_{t-1} \quad (9)$$

In this research work, DCC(m, n)-MGARCH(1,1) model along with different specifications of m and n is used.

Estimation: In estimation, the parameters of DCC-MGARCH model are estimated by keeping in mind the law of parsimony. To estimate the parameters one distribution for the standard errors z_t is considered. When z_t are multivariate Gaussian distributed, the joint distribution of z_t, \dots, z_T is:

$$f(z_t) = \prod_{t=1}^T \frac{1}{(2\pi)^{n/2}} \exp \left\{ -\frac{1}{2} z_t^T z_t \right\}$$

Since $E[z_t] = 0$ and $E[z_t z_t^T] = I$. by the use of rule for linear transformation of variables, the log-likelihood function for $a_t = H_t^{1/2} z_t$ is:

$$L(\theta) = \prod_{t=1}^T \frac{1}{(2\pi)^{n/2} |H_t|^{1/2}} \exp \left\{ -\frac{1}{2} a_t^T H_t^{-1} a_t \right\} \quad (10)$$

Where θ denotes the parameters of the model.

$$\begin{aligned} \ln(L(\theta)) &= -\frac{1}{2} \sum_{t=1}^T (n \ln(2\pi) + \ln(|H_t|) + a_t^T H_t^{-1} a_t) \\ &= -\frac{1}{2} \sum_{t=1}^T (n \ln(2\pi) + \ln(|D_t R_t D_t|) + a_t^T D_t^{-1} R_t^{-1} D_t^{-1} a_t) \\ &= -\frac{1}{2} \sum_{t=1}^T (n \ln(2\pi) + 2 \ln(|D_t|) + \ln(|R_t|) + a_t^T D_t^{-1} R_t^{-1} D_t^{-1} a_t) \end{aligned} \quad (11)$$

The accuracy is particularly difficult to obtain precise estimates of parameters such as the log-likelihood so to overcome this difficulty DCC-GARCH model is designed. The estimation is comprising of two stages, firstly we take the likelihood of R_t concerning the identity matrix I_n . Secondly, by taking the loglikelihood the parameters ψ are estimated.

$$\begin{aligned}
 \ln(L_1(\phi)) &= -\frac{1}{2} \sum_{t=1}^T (n \ln(2\pi) + 2 \ln(|D_t|) + \ln(|I_t|) + a_t^T D_t^{-1} I_n D_t^{-1} a_t) \\
 &= -\frac{1}{2} \sum_{t=1}^T (n \ln(2\pi) + 2 \ln(|D_t|) + a_t^T D_t^{-1} I_n D_t^{-1} a_t) \quad (12) \\
 &= -\frac{1}{2} \sum_{t=1}^T \left(n \ln(2\pi) + \sum_{i=1}^n [n \ln(h_{it}) + \frac{a_{it}^2}{h_{it}}] \right) \\
 &= \sum_{t=1}^T \left(-\frac{1}{2} \sum_{i=1}^n [n \ln(h_{it}) + \frac{a_{it}^2}{h_{it}}] + \text{constant} \right)
 \end{aligned}$$

The function for two-stage quasi-likelihood is then written as:

$$\begin{aligned}
 \ln(L_2(\psi)) &= -\frac{1}{2} \sum_{t=1}^T (n \ln(2\pi) + 2 \ln(|D_t|) + \ln(|R_t|) + a_t^T D_t^{-1} R_t^{-1} D_t^{-1} a_t) \\
 &= -\frac{1}{2} \sum_{t=1}^T (n \ln(2\pi) + 2 \ln(|D_t|) + \ln(|R_t|) + \varepsilon_t^T R_t^{-1} \varepsilon_t) \quad (13)
 \end{aligned}$$

when we apply condition on parameters from step one this yields a constant D_t , which can be excluded, and the constant terms can maximize as:

$$\ln(L_2^*(\psi)) = -\frac{1}{2} \sum_{t=1}^T (\ln(|R_t|) + \varepsilon_t^T R_t^{-1} \varepsilon_t)$$

Under certain conditions, it can be demonstrated that the pseudo-maximum-likelihood method gives us consistent and asymptotically normal estimators. We can consider full MLE as well as the two-step procedure which provides the uniform results.

Selection of Model

We select a model from the entire candidate's model which is simplest and gives all the information about the data based on Akaike Information Criterion (AIC) and Schwarz Bayesian Information Criterion (SBIC) by keeping in mind the law of parsimony.

Forecasting and its Evaluation

To forecast the matrix H_t of the DCC model, D_t from UGARARCH and R_t are computed separately, while the correlation coefficient is not itself forecasted but it is obtained as the ratio of forecasted covariance to the square root of the product of the forecasted variances.

Keeping in view the fact that a much larger number of forecasts is needed to examine the distribution of the forecasts error, 40% ($t=1, 2, \dots, 292$) of the observations was reserved as a fit period, while the remaining 60% ($t=1, 2, \dots, 436$) was reserved for the test period. One-step-ahead forecasts for conditional standard deviations and conditional correlation were generated using the rolling window technique (the window size was fixed at 292).

The model producing good out-of-sample forecasts will be used to generate the forecasts for the future values i.e., values after the sample period (after the last week of the data set understudy). The best forecasting model will be selected based on the following criteria:

$$MSFE = \frac{1}{T} \sum_{i=1}^T (\hat{h}_t^2 - h_t^2)^2 \quad (14)$$

$$RMSFE = \sqrt{\frac{1}{T} \sum_{i=1}^T (\hat{h}_t^2 - h_t^2)^2} \quad (15)$$

$$MAE = 100 \times \left[\frac{1}{T} \sum_{i=1}^T |\hat{h}_t^2 - h_t^2| \right] \quad (16)$$

RESULTS/FINDINGS AND DISCUSSIONS

Descriptive Statistics

(DAX), Pakistan Stock Exchange (PSX), and US Stock Market (NYSE) in US\$. Just looking at the values on vertical axes of prices one can easily observe that the Pakistani stock market has more variability as contrasted with the remaining ones (Lal, 2019). All the time series show a positive and increasing trend which is ultimately an indication of non-stationarity (i.e., mean & variance are time-dependent) in the prices. However, the behavior of three different Stock Markets is same except during the global financial crisis of 2007-2008 as PSX show relatively less variability as

compared to the other two markets. The movement of three different stock markets clearly shows the markets are co-integrated means all three stock markets are moving in the same direction.

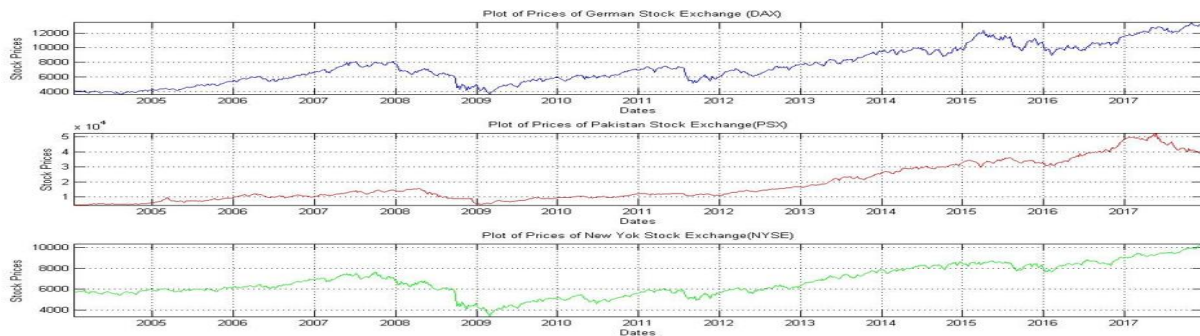


Figure 1: Weekly stock prices of different stock markets (DAX, PSX, and NYSE): January 2004 – December 2017

Figure 1 shows the time plot of weekly stock price indices of three different stock markets i.e., the German Stock Exchange.

Table 1: Summary statistics of weekly stock prices of DAX, PSX and NYSE

| Statistics | DAX | PSX | NYSE |
|----------------|------------------|------------------|------------------|
| Min. | 3646.99 | 4684.120 | 3364.65 |
| Max. | 13478.86 | 52636.871 | 10378.75 |
| Mean | 10378.75 | 18594.550 | 6717.191 |
| Variance | 616441 0 | 156535198.5 | 2117708 |
| Skewness | 0.528 | 0.984 | 0.338 |
| Kurtosis | 2.314 | 2.710 | 2.325 |
| ADF Test | 1.3668 (0.9572) | 2.154 (0.9925) | 1.3134 (0.9525) |
| Ljung-Box Test | 17466.28 (0.000) | 19412.49 (0.000) | 17798.36 (0.000) |

Note: ** shows the significance at 1% (i.e., $\alpha = 0.01$)

Likewise, Table 1 shows descriptive statistics and results of various tests utilized for checking the presence of serial correlation (Ljung-Box test), and non-stationarity (ADF-Test). The value of three different stock markets demonstrates the greater variability present in the weekly prices of the stock market. Similarly, the value of Skewness of three different stock markets and kurtosis display that price distributions of stock markets under study are away from normality.

As stated earlier, stock prices indicate a significant trend which is evidence of non-stationary structure in the time series data. Based on the p-values of the ADF test statistic, the null hypothesis in the Augmented Dickey-Fuller test is not rejected which means that all series have a unit root. Similarly, test statistics and the p-values of the Ljung-Box test show the presence of highly significant autocorrelation in the prices of all the stock markets under consideration.

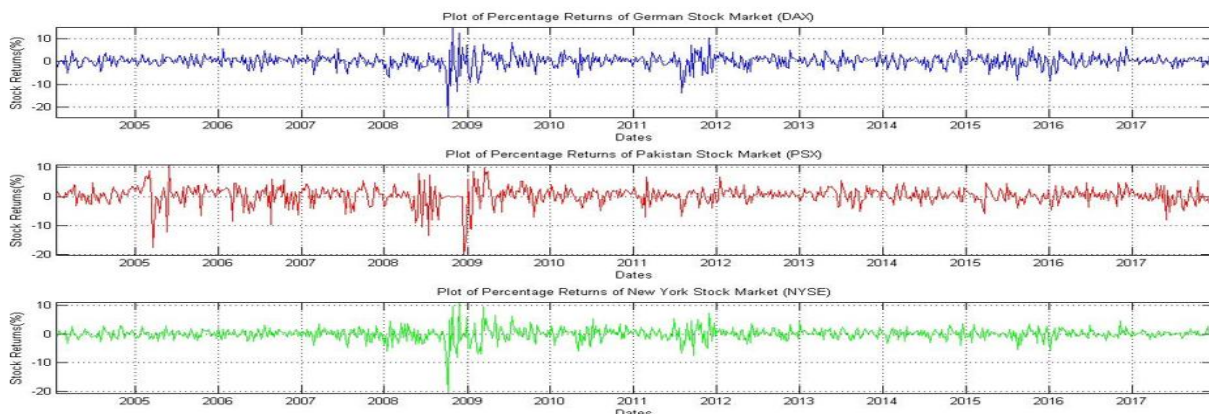


Figure 2: Weekly percent log-returns of German, Pakistan, and US stock markets

Since percent log-returns are calculated after taking the first difference of the prices hence make the series stationary (i.e., mean and variance of the data under study are now time-invariant). This statement can be verified from the results of the ADF test which are shown in figure 2. Furthermore, it can be observed from the above figure that clusters of low

and high volatility are existing in all stock returns which is an indication that the GRACH model can be used to capture this volatility clustering. We can perceive the effects of the financial crisis (2008) on all stock markets. The returns of DAX and NYSE follow the same pattern while the returns from PSX show a slight resemblance with the other two markets which is evidence of the high correlation between DAX and NYSE and the low correlation of PSX with these markets.

Table 2: Summary statistics of weekly percent log returns

| | DAX | PSX | NYSE |
|----------------|-----------------|-----------------|-----------------|
| Min | -0.2435 | -0.201 | -0.2061 |
| Max | 0.149422 | 0.109173 | 0.111828 |
| Mean | 0.001572 | 0.002962 | 0.000819 |
| Variance | 0.000875 | 0.000933 | 0.000491 |
| Skewness | -1.0236 | -1.4025 | -1.0863 |
| Kurtosis | 11.61402 | 9.629267 | 15.51559 |
| ADF-Test | -29.211 (0.001) | -23.632 (0.001) | -29.057 (0.001) |
| Ljung-Box Test | 53.573 (0.005) | 49.452 (0.014) | 55.524 (0.003) |

Table 2 shows the descriptive statistics of continuously compounded percent log returns of different stock markets. The mean returns of all the stock markets are nearly zero. Three return series are negatively skewed and at the same time having kurtosis values higher than the Gaussian distribution.

Figure 3 shows the ACFs and PACFs of squared returns of different stock markets. The large significant spikes at different lags are evident of the presence of the ARCH/GARCH effects and volatility clustering is present in the stock markets as well. The presence of volatility clustering was checked through Engle-ARCH and Mc-Leod and Li tests.

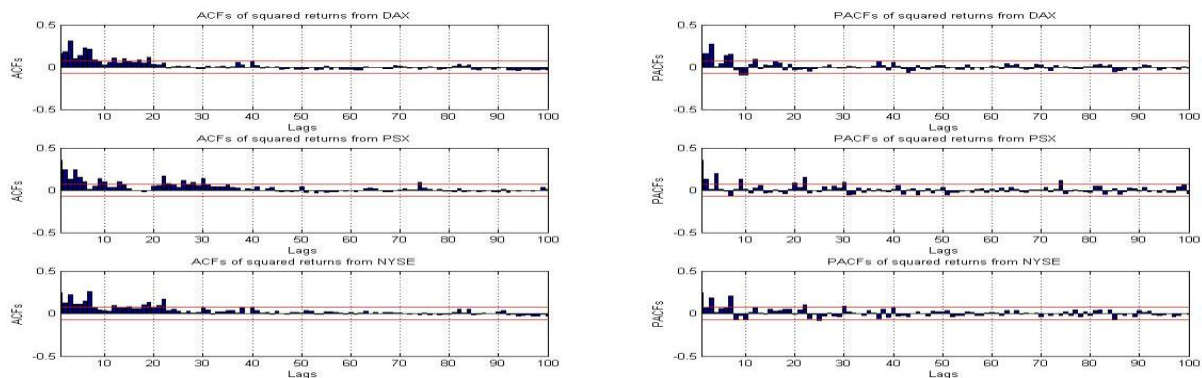


Figure 3: ACFs and PACFs of squared stock returns

The results for these tests are presented in Table 3 in which “H” being 1 indicates rejecting of the null hypothesis that there are no ARCH effects which can also be translated in terms of the confirmation of Heteroscedasticity. The smallest p-values of both test statistics indicate that squared returns are demonstrating the presence of volatility clustering and hence GARCH models in multivariate settings can be used to account for this volatility clustering phenomenon.

Table 3: Results of Engle ARCH test and McLeod Li Test for squared stock returns

| Stocks | Engle ARCH Test | McLeod Li Test |
|--------|-------------------|-------------------|
| | Q-Stat (p-value) | Q-stat (p-value) |
| DAX | 138.748 (0.000**) | 264.067 (0.000**) |
| PSX | 163.352 (0.000**) | 325.434 (0.000**) |
| NYSE | 134.382 (0.000**) | 267.006 (0.000**) |

Critical value at 0.05 is 31.4103 (** significantly shows rejection of null hypothesis)

Table 4 reports the unconditional correlations among three different stock market returns under study. As expected, there is a positive correlation between stock markets. A strong positive correlation (0.813) is observed between DAX and NYSE. Furthermore, PSX is showing weak positive correlations with the other two markets i.e., DAX and NYSE. Although the correlation of PSX is weak but highly significant this is an indication that markets under study move in the same direction.

Table 4: Lower triangular correlation matrix of different stock returns

| | DAX | PSX | NYSE |
|------|-----------------|-----------------|-----------|
| DAX | 1 (1.000) | | |
| PSX | 0.115 (0.002**) | 1 (1.000) | |
| NYSE | 0.813 (0.000**) | 0.107 (0.004**) | 1 (1.000) |

Note: **showing significance at the two-tailed correlation

Figure 4 shows time-varying conditional correlations between Pakistan & international (US & German) stock markets. One can easily observe a low conditional correlation of the Pakistan stock market with the other two markets while a high correlation was observed between the US and German stock markets. It can be seen clearly from the data in the graphic that despite the low correlation of PSX between the other two markets (DAX, and NYSE), the co-movements between Pakistan & International stock markets are positive.

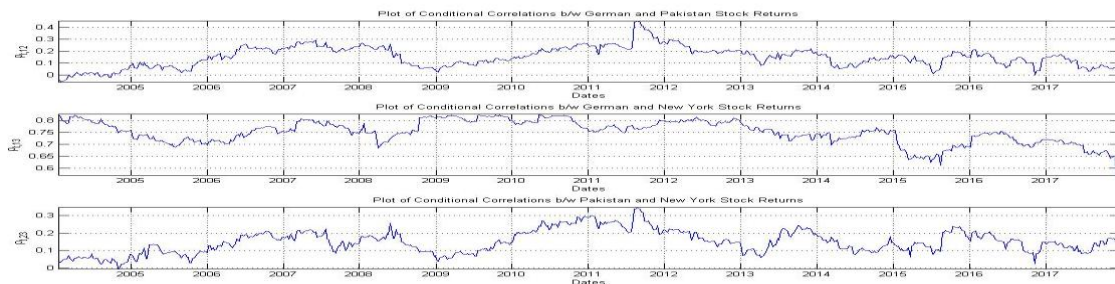


Figure 4: Time-varying conditional correlations of stock markets from DCC model

Since the DCC-MGARCH model has the capability to model conditional correlations and conditional variances simultaneously, so besides plotting of conditional correlations, the conditional variances as captured by the GARCH model are plotted in the following figure (i.e., Figure 5). This figure clearly shows that there is large volatility present in Pakistan Stock Market (PSX) whereas the other two stock markets are less volatile as compared to PSX. From this figure, one can observe that German & US stock market follows the same pattern. We can also observe a sharp increase in volatility during the financial crises of 2008 in all stock markets. Pakistan's stock market has also experienced high volatility during the financial crises of 2008.

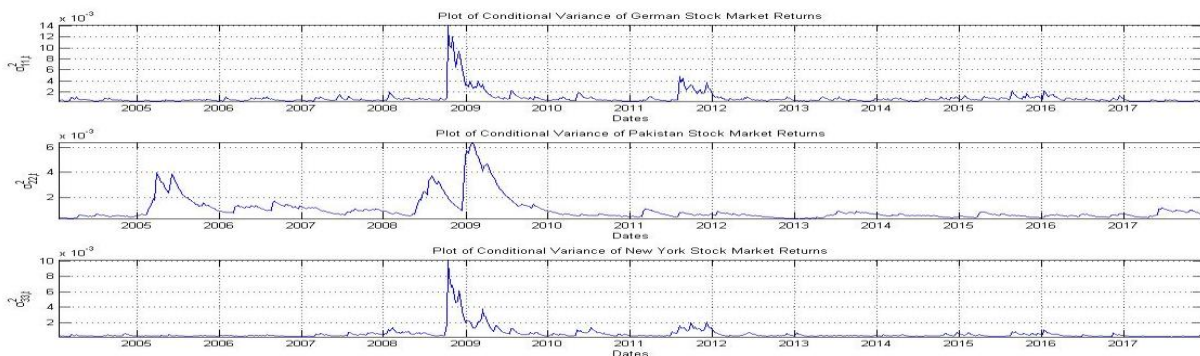


Figure 5: Plot of conditional variances of different stock markets

To capture the volatility clustering present in the stock markets under study, DCC-MGARCH models with different specifications (i.e. with different lag orders) were estimated and the most suitable was selected based on maximized log-likelihood function, the information criteria such as AIC, BIC, HQC, and also on the white noise property of the residuals from the fitted models (Andersson-Shall and Lindskog, 2019).

Table 5: Parameter estimates of DCC(1,1)-GARCH(1,1) Model

| Stocks | α_0 | α_1 | β_1 | r_{ij} | α | β |
|------------|------------|------------|------------|------------|------------|------------|
| DAX | 0.000 | 0.226 | 0.696 | 0.136 | | |
| (t-values) | (23941.38) | (6.820562) | (19.64591) | (78.63645) | | |
| PSX | 0.000 | 0.076 | 0.911 | 0.751 | 0.019 | 0.960 |
| (t-values) | (79474.11) | (160.4823) | (983.8221) | (138.5728) | (468.6538) | (4095.232) |
| NYSE | 0.000 | 0.208 | 0.734 | 0.146 | | |
| (t-values) | (8991.74) | (16.09246) | (47.895) | (140.4265) | | |

Table 5 shows the parameter estimates of the selected DCC (1,1)-GARCH (1,1) model. However, the values of LL, AIC, BIC, HQC, and the Q-statistics and p-values of Ljung-Box test of different DCC(p,q)-GARCH(1,1) models are reported in Table 06. Based on t-values (reported in parenthesis), we can say that all the parameters are highly significant. It can be easily observed from the parameters of the GARCH(1,1) model that high persistence in volatility ($sum\ of\ \alpha_1\ \&\ \beta_1$) is existing in PSX followed by NYSE and DAX. Besides, the parameters of the DCC model are also evidence of high persistence of conditional correlations among the three markets under consideration (Bala and Takimoto, 2017).

The model selection is purely based on the smallest Akaike Information Criterion (AIC), Schwarz Bayesian Information Criterion (SBIC), and Hannan-Quinn Information Criterion (HQC) values, and the results of the Ljung-Box test are presented in the above table. Besides these three criteria, the residuals of the selected model should behave like a white noise process (without having any significant autocorrelations in the residuals of the selected model). Based on the above table log-likelihood method selects the DCC (3, 3) model, which is a model with a high number of parameters. It is well documented in the literature that the log-likelihood method always selects the model with a high number of parameters (Swiler et al., 2011).

Table 6: Model Selection based on LL, AIC, BIC, HQC and Ljung-Box Test

| Model | LL | AIC | SBIC | HQC | Ljung-Box Test |
|----------|---------|-----------|-----------|-----------|----------------|
| DCC(1,1) | 5389.04 | -10750.00 | -10686.00 | -10725.00 | 29.738 (0.09) |
| DCC(1,2) | 5389.30 | -10749.00 | -10680.00 | -10722.00 | 28.940 (0.11) |
| DCC(1,3) | 5392.90 | -10754.00 | -10680.00 | -10725.00 | 28.010 (0.13) |
| DCC(2,1) | 5389.00 | -10748.00 | -10679.00 | -10722.00 | 27.320 (0.29) |
| DCC(2,2) | 5389.30 | -10747.00 | -10673.00 | -10718.00 | 27.020 (0.36) |
| DCC(2,3) | 5392.90 | -10752.00 | -10674.00 | -10722.00 | 26.181 (0.43) |
| DCC(3,1) | 5390.20 | -10748.00 | -10675.00 | -10720.00 | 25.920 (0.49) |
| DCC(3,2) | 5390.20 | -10746.00 | -10668.00 | -10716.00 | 25.450 (0.52) |
| DCC(3,3) | 5393.20 | -10750.00 | -10668.00 | -10719.00 | 24.705 (0.63) |

In the same way, Akaike Information Criterion (AIC) selects DCC (1, 3), which is a model with the second high number of parameters. So now we see the results of Schwarz Bayesian Information Criterion (SBIC), SBIC selects the DCC (1,1) model, which is a model with the lowest number of parameters. Similarly, Hannan-Quinn Information Criterion (HQC) selects the two models DCC (1,1) and DCC (1,3) simultaneously. Based on the argument that we always need to select the parsimonious (with a smaller number of significant parameters) model; hence, DCC (1,1) model was selected (Fan and Wang, 2008).

Forecasting from DCC (1,1)-GARCH (1,1) model

The real test of the DCC-MGARCH (p, q) model family lies in the forecasting. The selected DCC (1,1)-GARCH (1,1) model was used to forecast the conditional correlations and conditional variances of weekly stock returns of different stock markets from the first week of January 2004 to the last week of December 2017 by using the observed data. The forecasted squared stock returns were compared with the observed squared stock returns to get the idea that how well our selected model performs in terms of forecasting.

Since we used out-of-sample evaluation method in forecasting, so we first split our data set into two parts i.e., the first part which consists of 40% (292 observations) of the total observation was used for estimation purpose while the remaining 60% (436 observations) was used for forecasting purpose. One-step-ahead forecasts were generated using the moving window technique. The forecasting errors were calculated using these forecasts and their corresponding observed values. For the forecast evaluation purpose, Mean Square Forecast Error (MSFE), Root Mean Square Forecast Error (RMSFE), and Mean Absolute Percentage Error were used. Table 7 reports the results of the MSFE, RMSFE, and MAPE resulted from different DCC-GARCH models.

Table 7: Out of sample forecast evaluation based on MSFE, RMSFE and MAPE

| Criteria | MSFE | RMSFE | MAPE |
|------------|--------|--------|--------|
| DCC (1, 1) | 0.0667 | 0.2582 | 3.1900 |
| DCC (1, 2) | 0.0671 | 0.2590 | 3.2000 |
| DCC (1, 3) | 0.0728 | 0.2669 | 4.7450 |
| DCC (2, 1) | 0.0319 | 0.1787 | 3.1950 |
| DCC (2, 2) | 0.0671 | 0.2590 | 3.2005 |
| DCC (2, 3) | 0.0524 | 0.2289 | 3.993 |
| DCC (3, 1) | 0.0756 | 0.2749 | 4.2575 |
| DCC (3, 2) | 0.0756 | 0.2750 | 4.2503 |

| | | | |
|------------|--------|--------|--------|
| DCC (3, 3) | 0.0431 | 0.2077 | 4.4098 |
|------------|--------|--------|--------|

It can be seen from Table 7 that all the out of sample forecasts criterion selects DCC (2,1) model. In the present study, in the case of in-sample-fitting and out-of-sample forecast evaluation, different models are selected. It is worth mentioned here that a model with the best in-sample fitting doesn't need to provide better out-of-sample forecasts (Fildes and Petropoulos, 2015).

Figures 6 and 7 show the plot of observed and forecasted conditional correlations and conditional variances from DCC (2,1)-GARCH (1,1) model, respectively. Our selected model produces forecasts which are slightly greater than the observed values which means that our model demonstrates an upward bias (Brunori et al., 2019). The uncertainty associated with each estimated value is greater than the expected and hence upward biased is introduced in the estimated model.

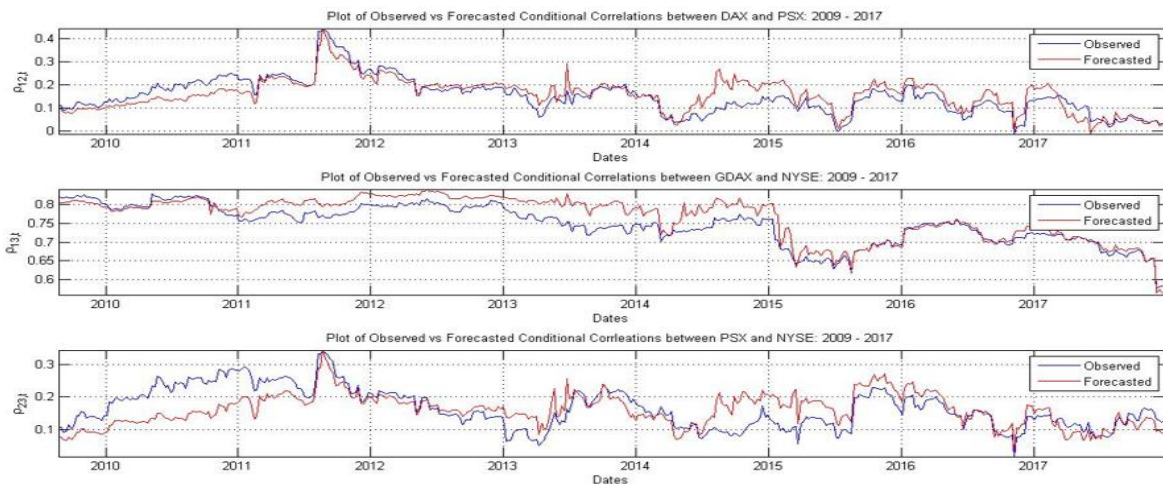


Figure 6: Plot of observed and forecasted conditionals correlations

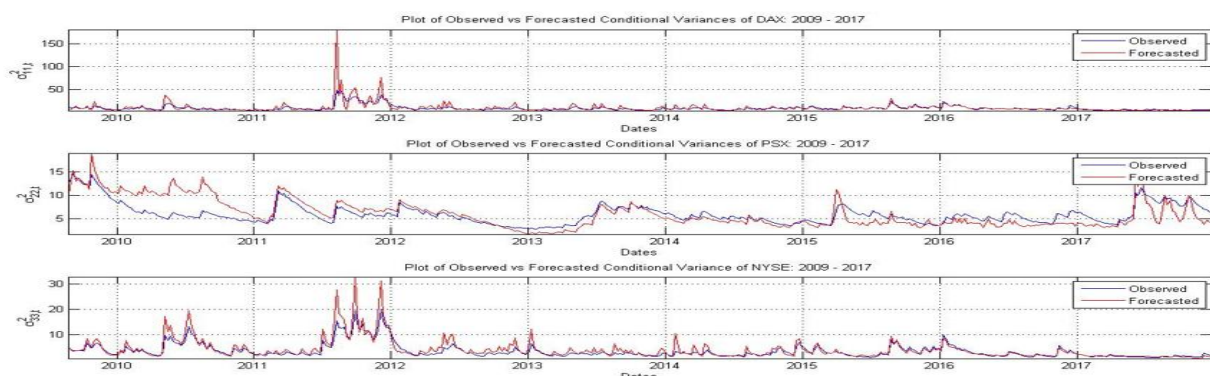


Figure 7: Plot of observed and forecasted Conditional Variances

CONCLUSION

The co-movement of the Pakistan stock market (PSX) in comparison with other selected countries' stock markets is more positive and progressive as it provides diverse opportunities to Pakistani as well as foreign investors. Moreover, the study found that the Pakistan stock market for local and foreign investors and fund managers is attention-grabbing for beneficial investments; because it reduces its stake in global affairs, which is caused by the low involvement of Pakistan in other developing-world affairs. It extends the opportunities of the various benefits since investors are searching such markets which have greater diversification paybacks, for the reason that diversification minimizes the risk element. Less co-movement will increase the foreign direct investment in Pakistan, which is much helpful for the development of the economy of Pakistan through investing in energy, communication & industrial sector, strengthening the Pakistani rupee, reducing the dependency on foreign debts (i.e., IMF & World Bank), reducing the imports of capital items and making the budget favourable. The results of this study are also equally beneficial for Pakistani investors and policymakers. Pakistani investors and fund managers can also receive the diversification benefits from the selected stock markets of the US and Germany.

SUGGESTIONS FOR FUTURE STUDIES

It is recommended that the weak correlations of the Pakistan stock exchange (PSX) with the other two markets suggest that weak stock market integration between these markets i.e. Pakistan stock market (PSX) is very attractive to investors to keep benefits from diversification keeping in view portfolio theory. Additionally, the high volatility in Pakistan Stock

Exchange (PSX) provides low investment opportunities so the government of Pakistan should devise new policies that weaken the volatility in the Pakistan stock market (PSX) to attract local and foreign investors. Last but not least the weak correlation between Pakistan and these two markets may be suggestive of different monetary policies leading to the different business cycle dynamics of the respective markets and the outcome is to erase the stock market correlations hence further studies are suggestive to be carried out in this regard.

ACKNOWLEDGEMENT

The authors are grateful to an anonymous referee and the editor for many helpful comments and suggestions. Any errors or omissions are, however, our own.

AUTHORS CONTRIBUTION

The main idea of this research was presented and devised by Naeem Ahmed Qureshi and Ali Akbar Pirzado. The theory portion was covered by Imran Khan Jatoi. The analytical methods were verified by Riaz Ali Buririo and Komal Aarain. Ali Akbar Pirzado and Naeem Ahmed Qureshi finalize the Analysis section, as well as results, were discussed by Ali Akbar Pirzado and Imran Khan Jatoi.

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