A deterministic inventory model for deteriorating products with shortages and price dependent demand rate

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Abstract

Purpose of the study: The proposed model shows a deterministic approach of the inventory management in which the rate of the deterioration of the inventory commodities is proportional to time, demand rate is a function of selling price and inventory holding cost both, ordering cost and deterioration rate are all function of time. Here shortages are allowed during the lead time and completely backlogged. The optimum replenishment policy is to be determined which minimizes the total cost. The rate of deterioration has been considered as non-instantaneous and that follows the two parameters Weibull distribution. In the proposed model we aim to find optimal value of the inventory level to minimize the total effective cost.

Methodology: The optimal solution will have to be obtained by using Mathematica Software and has been illustrated using a numerical example.

Main Findings: Since lowering the selling price would result in increase in purchase of items by the customers, hence the selling price is the major criteria to determine their buying capacity.

Applications of this study: If researchers go on taking several kinds of variable costs etc., then hopefully they may find a new area of study in their research.

INTRODUCTION

Usually it has been seen that in a big market a large quantity of goods leads to the customers to buy more goods, thereby resulting in the creation of greater demand of the goods, which motivates the retailer to increase more quantity, but keeping deterioration of the inventory items in mind when management is to decide when and how much to order or to manufacture so as to keep total cost associated with it as the minimum. Certain products such as Vegetables, medicine, gasoline, blood and radioactive chemical all decay during their normal storage period and as such, while determining the optimal inventory policy of such type of items, the loss due to deterioration cannot be ignored. The EOQ model is the oldest model and lot of work has been done on this model. In this classical model the demand rate is assumed to be constant. But in practice demand may be time-dependent, stock-dependent & price dependent. Hence selling price has great importance in an inventory management system. In the past decades the study of deteriorating items has gained enormous popularity and importance. Ghare and Schrader were first to use the concept of deterioration and they developed their inventory model with a constant rate of deterioration i.e., a model for exponential decaying inventory. Ghare and Schrader (1963) followed by Covert, R. P. and Philip, G.C (1973) who formulated a model using a variable rate of deterioration with two parameter Weibull distribution which was later on extended by Pal, S., Goswami, A., and Chaudhari, K.S. (1993). Most of the authors viz., Ben-daya, M. and Abdul, R. (1994), Naddor (1966), Das (1975), Duari, N.K. & Chakroborti, T. (2014), You, S. P. (2005), Chung, K., and Ting, P(1993), assumed lead time as prescribed in all cases i.e., deterministic as well as probabilistic. Lead time can be reduced at an added cost in practice so by reducing the lead time customer service and responsiveness pertaining to production schedule changes can be improved and thus reduction in safety stock can be achieved.

Chen, L.H., & Kang, F.S., (2009 a) formulated the integrated inventory models under the two -level trade- credit policy with price - negotiation scheme. A review article on trade credit can be reviewed by Cheng et. al, (2009) incorporated the concept of vendor and buyer considering trade credit and items of imperfect quality. Chang, H., Ho, C., Ouyang, L., Su, C. (2009) gave the models discussing the optimal pricing and ordering policy for an integrated inventory model when trade credit linked to order quantity. Aggarwal, S. P., & Jaggi, C. K. (1995) discussed the ordering policies of deteriorating items under permissible delay in payments. Teng, J.T., Chang, C.T. and Chern, M.S. (2012) proposed a Vendor-buyer inventory models with trade credit financing under both non-cooperative and integrated environments.

The prescribed model gave an analytical solution with the help of a numerical example.

Notations and Assumptions

Notations: The following notations were used while developing the model:-

$\theta(t)$:   Time dependent deterioration rate,

D(p):   Demand rate,

l: Lead Time,

a_m: Amount of deteriorated materials during a cycle time,

Q: Maximum Inventory level,

c_i: The unit cost per item,

o_c: The ordering cost of inventory per order,

T_1:   Replenishment cycle time,

p_c: Purchase cost,

S_c: Shortage cost,

h_c: Total holding cost,

h: The inventory holding cost per unit item per unit time,

$Q'(t)$:   The inventory level at any instant of time t,

c_d: Total deterioration cost per cycle,

T: A cycle time.

Assumptions

Following are the assumptions adopted in the proposed model:-

1. The rate of deterioration $\theta(t), t > 0$ follows the two parameters Weibull distribution i.e., $\theta = \alpha\beta t^{(\beta-1)}$, where $\alpha(0 < \alpha < 1)$ is the scale and $(\beta > 0)$ is the shape parameter,

2. The demand rate $D(p)$ is price sensitive and is given by $D(p) = \gamma t^{-\delta}; \gamma, \delta > 0$ & p is selling price,

3. The ordering quantity is given by $Q + lD(p)$; where, $t = l$,

4. The cost of items remains the same and does not depend on the order size of the item,

5. The deterioration of items are not instantaneous,

6. There is no repair or replenishment of the deteriorating inventory cycle,

7. Single Warehouse has been considered,

8. The inventory is replenished only once in each cycle,

9. Holding cost is time-dependent,

10. Rate of replenishment is infinite,

11. Lead time is constant.

12. Shortages are allowed during the lead time.

MATHEMATICAL MODEL AND ANALYSIS

Since the demand is price dependent and the depletion of inventory level takes place due to both the demand (supply) as well as the deterioration in each cycle. Our Objective is to determine the optimal ordering quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible, where the holding cost is time-sensitive and shortages are allowed during lead time. Thus shortages takes place and accumulate to the level (say) s_1 at any instant of time $t = T$. The behavior of inventory system at any time is depicted by the figure 1 below.
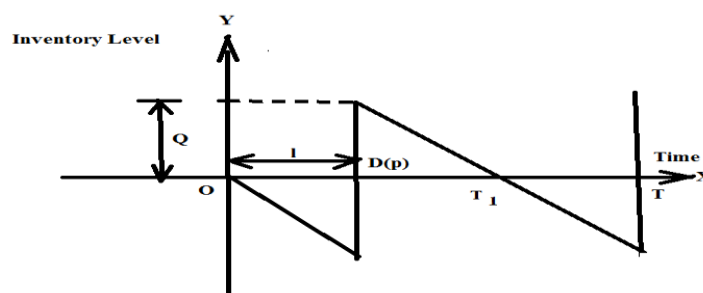

Figure 1: Graphical representation of the inventory system

The states of Inventory level $Q'(t)$ at any cycle time T is governed by the following set of 1st order differential equations:

$$\frac{dQ'(t)}{dx} + \alpha\beta(t)^{\beta-1}Q'(t) = -\gamma p^{-\delta}; l \le t \le T_1 \qquad \ldots\ldots (1)$$

$$\frac{dQ'(t)}{dx} = -\gamma p^{-\delta}; T_1 \le t \le T \qquad \ldots\ldots (2)$$

with the boundary condition $Q'(T_1) = 0$; $Q'(T) = s_1$ (say). The solution of the above differential equations using boundary conditions results in,

$$Q'(t).e^{\alpha(t)^\beta} = -\gamma p^{-\delta}\left[t + \alpha\frac{t^{\beta+1}}{\beta+1}\right] + A; \qquad \ldots\ldots (3)$$

where, A is constant of integration.

Solving eqn. (1), we obtain $Q'(t)$ during the time period $(l \le t \le T_1)$ as

$$Q'(t) == -\gamma p^{-\delta}\left[(T_1 - t) + \alpha\frac{(T_1^{\beta+1}-T^{\beta+1})}{\beta+1}\right]\left[1 - \alpha(t)^\beta\right]; l \le t \le T_1 \qquad \ldots\ldots (4)$$

Now at time t=l, $Q'(l) = Q$ i.e., when the items are received initially, the inventory level is maximum and hence, from the eqn. (4) Q is obtained, where $Q + lD(p)$ is the ordering quantity at the beginning of the cycle. Since we have considered that the lead time is constant, the items are received non-instantaneously.

Therefore, $Q === \gamma p^{-\delta}\left[(T_1 - l) + \alpha\frac{(T_1^{\beta+1}-l^{\beta+1})}{\beta+1}\right]\left[1 - \alpha(l)^\beta\right]; \qquad \ldots\ldots (5)$

At $t = T$, $s_1 = -\gamma p^{-\delta}(T - T_1)$, since $Q'(t) = -s_1$ $\qquad \ldots\ldots (6)$

The extent to which the materials deteriorated in one complete cycle is given by

$$a_m(t) = \gamma p^{-\delta}\left[(T_1 - l) + \alpha\frac{(T_1^{\beta+1}-l^{\beta+1})}{\beta+1}\right]\left[1 - \alpha(l)^\beta\right] - D(T_1 - l) \qquad \ldots\ldots (7)$$

Now the total variable cost includes:

(a) The ordering cost (which is considered to be the same per order during the current financial year)

(b) The deterioration cost is given by $c_i.a_m$, which is thus,

$$c_d = c_i\left[\gamma p^{-\delta}\left[(T_1 - l) + \alpha\frac{(T_1^{\beta+1}-l^{\beta+1})}{\beta+1}\right]\left[1 - \alpha(l)^\beta\right] - D(T_1 - l)\right] \qquad \ldots\ldots (8)$$

(c) The holding cost is dependent on the average inventory cost, it is given by

$\int_l^{T_1} h\,Q'(t)dt$, simplifying which we obtain

$$h_c = h\,\gamma p^{-\delta}\left[\left[\left(\frac{T_1^2}{2} - \alpha\frac{T_1^{\beta+2}}{\beta+1} + \alpha\frac{T_1^{2\beta+2}}{\beta+1} + \alpha\frac{T_1^{2\beta+2}}{\beta+2}\right) + \frac{\alpha}{\beta+1}\left(\begin{array}{c}T_1^{\beta+1} - \frac{T_1^{\beta+2}}{\beta+2} - \\ \alpha\frac{T_1^{2\beta+2}}{\beta+1} + \alpha\frac{T_1^{2\beta+2}}{\beta+2}\end{array}\right)\right] - \left(lT_1^{\beta+1} - \frac{l^{\beta+2}}{\beta+2} - \alpha\frac{(T_1^{\beta+1}l^{\beta+1})}{\beta+1} + \alpha\frac{l^{2\beta+2}}{2\beta+2}\right)\right]$$

$$\ldots\ldots (9)$$

(d) The purchase cost is given by :-

$\quad p_c = c_i[Q + lD(p)]$

$$p_c = c_i\left[\gamma p^{-\delta}\left\{(T_1 - l) + \alpha\frac{(T_1^{\beta+1}-l^{\beta+1})}{\beta+1}\right\}\left[1 - \alpha(l)^\beta\right] + l\gamma p^{-\delta}\right] \qquad \ldots\ldots (10)$$

(e) Shortage cost is given by

$$s_c = s\left[-\int_T^{T_1} Q\,Q'(t)dt\right]$$

$$s_c = \left[s\gamma p^{-\delta}\left(\frac{T^2+T_1^2}{2} - TT_1\right)\right] \qquad \ldots\ldots (11)$$

Now the total variable cost function T_c for one complete cycle is given by

$$T_c = o_c + c_p + h_c + c_d$$

Hence, the total cost is

$$T_c = o_c + c_i \left[\gamma p^{-\delta} \left\{ (T_1 - l) + \alpha \frac{(T_1^{\beta+1} - l^{\beta+1})}{\beta+1} \right\} [1 - \alpha(l)^{\beta}] + l\alpha p^{-\delta} \right]$$

$$+ h \gamma p^{-\delta} \left[\left\{ \left(\frac{T_1^2}{2} - \alpha \frac{T_1^{\beta+2}}{\beta+1} + \alpha \frac{T_1^{2\beta+2}}{\beta+1} + \alpha \frac{T_1^{2\beta+2}}{\beta+2} \right) + \frac{\alpha}{\beta+1} \left(\begin{array}{c} T_1^{\beta+1} - \frac{T_1^{\beta+2}}{\beta+2} - \\ \alpha \frac{T_1^{2\beta+2}}{\beta+1} + \alpha \frac{T_1^{2\beta+2}}{\beta+2} \end{array} \right) \right\} - \left(lT_1^{\beta+1} - \frac{l^{\beta+2}}{\beta+2} - \alpha \frac{(T_1^{\beta+1} l^{\beta+1})}{\beta+1} + \alpha \frac{l^{2\beta+2}}{2\beta+2} \right) \right]$$

$$+ c_i \left[\gamma p^{-\delta} \left[(T_1 - l) + \alpha \frac{(T_1^{\beta+1} - l^{\beta+1})}{\beta+1} \right] [1 - \alpha(l)^{\beta}] - D(T_1 - l) \right] \left[s\gamma p^{-\delta} \left(\frac{T^2 + T_1^2}{2} - TT_1 \right) \right]$$

……. (12)

We have to find optimal value of Q to minimize T_c. Hence the values of T_1 for which $\frac{\partial T_c}{\partial T_1} = 0$, satisfying the condition $\left(\frac{\partial^2 T_c}{\partial T_1^2} \right) > 0$

Therefore, the optimal solution of the equation (12) is obtained by using Mathematica Software and has been illustrated using the following numerical example: -

NUMERICAL EXAMPLE

If we consider the following values of the parameters i.e., $h = 5$,

$\gamma = 6$, $s = 1, T = 1\, Yr., \alpha = 0.005$, $\beta = 0.4$, $a_m = 300, c_i = 9, l = 7, D = 50, p = 12$, then we get the optimal value to be $Q = 33.15$, $T_1 = 72.6$ days and minimum total cost as $T_c = 3389.73$ per year.

CONCLUSION

In this paper we adopted this model to show that the demand rate is a function of selling price. Since lowering the selling price would result in increase in purchase of items by the customers, hence the selling price is the major criteria to determine their buying capacity. Here shortages are allowed during the lead time and completely backlogged. There is still enough room for researchers to work on taking several kinds of variable costs etc.

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