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MULTIPLE DICTIONARY FOR SPARSE MODELING

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Abstract— **Much of the progress made in image processing in the past decades can be attributed to better modeling of image content, and a wise deployment of these models in relevant applications. In this paper, we review the role of this recent model in image processing, its rationale, and models related to it. As it turns out, the field of image processing is one of the main beneficiaries from the recent progress made in the theory and practice of sparse and redundant representations. Sparse coding is a key principle that underlies wavelet representation of images. Sparse representation based classification has led to interesting image recognition results, while the dictionary used for sparse coding plays a key role in it. In general, the choice of a proper dictionary can be done using one of two ways: i) building as parsifying dictionary based on a mathematical model of the data, or ii) learning a dictionary to perform best on a training set.**

I. INTRODUCTION

The process of digitally sampling a natural signal leads to its representation as the sum of Delta functions in space or time. This representation, while convenient for the purposes of display or playback, is mostly inefficient for analysis tasks. Signal processing techniques commonly require more meaningful representations which capture the useful characteristics of the signal for recognition, the representation should highlight salient features; for denoising, the representation should efficiently separate signal and noise; and for compression, the representation should capture a large part of the signal with only a few coefficients. Interestingly, in many cases these seemingly different goals align, sharing a core desire for simplification. Representing a signal involves the choice of a dictionary, which is the set of elementary signals or atoms used to decompose the signal. When the dictionary forms a basis, every signal is uniquely represented as the linear combination of the dictionary atoms. In the simplest case the dictionary is orthogonal, and the representation coefficients can be computed as inner products of the signal and the atoms; in the non-orthogonal case, the coefficients are the inner products of the signal and the dictionary inverse, also referred to as the bi-orthogonal dictionary.

In the last decade, sparsity has emerged as one of the leading concepts in a wide range of signal-processing applications (restoration, feature extraction, compression, to name only a few applications). Sparsity has long been an attractive theoretical and practical signal property in many areas of applied mathematics (such as computational harmonic analysis, statistical estimation, and theoretical signal processing).The sparse representation theory has shown that sparse signals can be exactly reconstructed from a small number of elementary signals (or atoms).The sparse representation of natural signals can be achieved by exploiting its sparsity or compressibility. A natural signal is said to be sparse signal if that can be compactly expressed as a linear combination of a few small number of basis vectors .Sparse representation has become an invaluable tool as compared to direct time-domain and transform-domain signal processing methods.Sparse and redundant representation modeling of data assumes an ability to describe signals as linear combinations of a few atoms from a pre-specified dictionary. As such, the choice of the dictionary that sparsifies the signals is crucial for the success of this model. Unlike to DCT, DWT and PCA analysis and their variations sparse models do not impose any condition of orthogonallity on the basis vectors, hence allowing more flexibility to adapt the representation.

II. PROPOSED SOLUTION

A. *Dictionary Learning:*

In its simplest form a linear generative model can be written algebraically as:

$$X = DS + \varepsilon = \sum_{i=0}^{n} d_i s_i + \varepsilon \qquad (1)$$

The input vector $X \in R^{m\times 1}$ is a vector of data derived from the image block under study, which is to be encoded. The observation is modeled as a linear combination of atoms d_i, which are the elementary structural representations establishing the dictionary $D^{m\times k}$. The vector S defines the multiplicative weights of these features. The vector ε represents the error due to the inability of the model to represent the input exactly. If the requirement is to have zero observation error, i.e. $\varepsilon = 0$ and a square known orthogonal matrix D, the problem of finding S for any input X becomes a standard orthogonal transform. For cases where both D and S are unknown for $m \geq k$, PCA, ICA and similar variations are the established tools, which guarantee to have the elements of S uncorrelated and independent. The central problem in applying PCA and ICA is in finding the matrix D while minimizing ε. Unlike to this sparse modeling deals with $m << k$.Apart from estimating the dictionary D the need is also to find a method to compute S for a given estimate. When a sufficient amount of training data is available, the dictionary can be adapted to the data itself. The joint optimization problem of dictionary learning and sparse coding is then expressed as:

$$\min_{D,S} \|X - DS\|^2 + \lambda \|s_i\|_p, \quad subject\ to\ \forall k, \|d_k\|_2 \leq 1 \quad \ldots\ldots\ldots\ldots\ldots(2)$$

where $\|s_i\|_p$ is the l_p norm and $0 \leq p \leq 1$ measures the sparsity of the vector s_i. Sparse approximation refers to an representation of X with fewer non-zero coefficients than the dimension m of X. A measure of the sparsity of a representation is counting the zeros or L_0 norm. This norm is easy to compute, however, the problem of finding the minimum of equation (2) with this norm is NP-hard . Different approximations to this norm have therefore been discussed.

A wide range of dictionary learning algorithms have been proposed in the literature . Standard unsupervised methods like K-means, K-SVD [1], and LLC [25] train visual dictionaries based on only information of images. A widely acceptable dictionary learning method for image restoration is the KSVD algorithm, which learns an over-complete dictionary from a training dataset of natural image patches. Statistical learning theory has proved that augmenting the amount of training samples dramatically enhances the generalization of dictionary. However, addition of training samples may also lead to enlarge the variation in pattern with appearance of many types of textures and edges. Apparently natural images being greatly flushed with smooth blocks, use of single dictionary don't efficiently preserve the high order statistical manifolds. Based on KSVD, Mairal added a discriminative reconstruction constraint in the DL model to gain the discriminative ability. Algorithms that incorporate class-specific information when learning the dictionary have also been developed, and successfully applied for digit recognition and image classification.

➢ K-SVD Algorithm

1. Initialize Dictionary
- Select atoms from input
- Atoms can be patches from the image
- Patches are overlapping

2. Sparse Coding(OMP)
- Use OMP or any other fast method
- Output gives sparse code for all signals
- Minimize error in representation

3. Update Dictionary-One atom at a time
- Replace unused atom with minimally represented signal
- Identify signals that use *k*-th atom (non zero entries in rows of X)
- Minimize this error matrix with rank-1 approx from SVD
- [U,S,V] = svd(E_k)
- Replace coeffiecient of atom d_k in X with entries of s_1v_1
- $d_k = u_1/\|u_1\|_2$

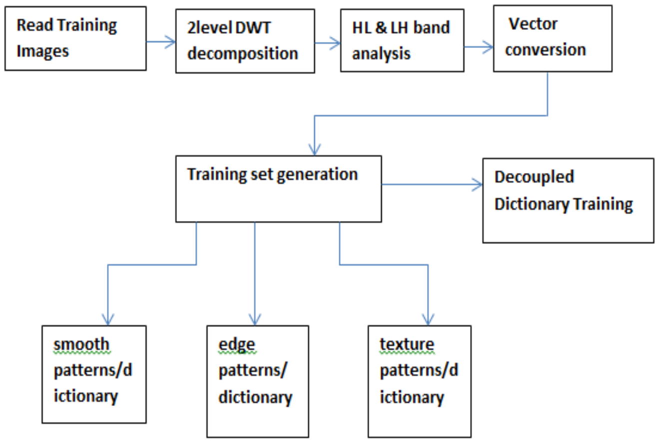
This paper addresses about learning of multiple decoupled dictionaries in a framework to use Discrete Wavelet Transform for pattern classification.

Sparse coding is the process of computing the representation coefficients X based on the given signal Y and the dictionary D. This process, commonly referred to as "atom decomposition," requires solving above equations , and this is typically done by a "pursuit algorithm" that finds an approximate solution. Exact determination of sparsest representations proves to be an NP-hard problem. Thus, approximate solutions are considered instead, and in the past decade or so several efficient pursuit algorithms have been proposed. The simplest ones are the matching pursuit (MP) and the orthogonal matching pursuit (OMP) algorithms. These are greedy algorithms that select the dictionary atoms sequentially. These methods are very simple, involving the computation of inner products between the signal and dictionary columns, and possibly deploying some least squares solvers. Both above equations are easily addressed by changing the stopping rule of the algorithm.

A second well-known pursuit approach is the basis pursuit (BP). It suggests a convexification of the problem sposed in the above equations by replacing the lo-norm with and l1-norm. The focal underdetermined system solver (FOCUSS) is very similar, using the l p-norm with p≤ 1 as a replacement for the lo –norm in the above equations. Sparse representation of signals have drawn considerable interest in recent years. Sparse coding is a key principle that underlies wavelet representation of images. In this paper, we explain the effort of seeking a common wavelet sparse coding of images from same object category leads to an active basis model called Sparse-land model, where the images share the same set of selected wavelet elements, which forms a linea rbasis for restoring the blurred image. The aim of image restoration is the removal of noise (sensor noise, blur etc) from images. The simplest approach for noise removal is based on various types of filters such as low-pass filters or median filters. In this paper, we used Haar wavelet transform based Weiner filter followed by PCD algorithm. We observed that the outcomes does not show any tendency to be sparse. In this case, we start by defining a random generator of sparse vector and it can be modified in various ways. One among them is MMSE(Minimum mean square error) estimator and it is used in this paper. This analysis gives a more solid foundation for the sparsest representation of deblurred image, which is said to be Sparse-land model.

B. Proposed Sparse Modeling Using DWT Classification And Decoupled Dictionary

I) Dictionary Training



II) Sparse Coding

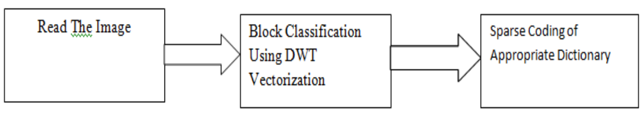


Fig:Sparse Coding

C. Block Diagram

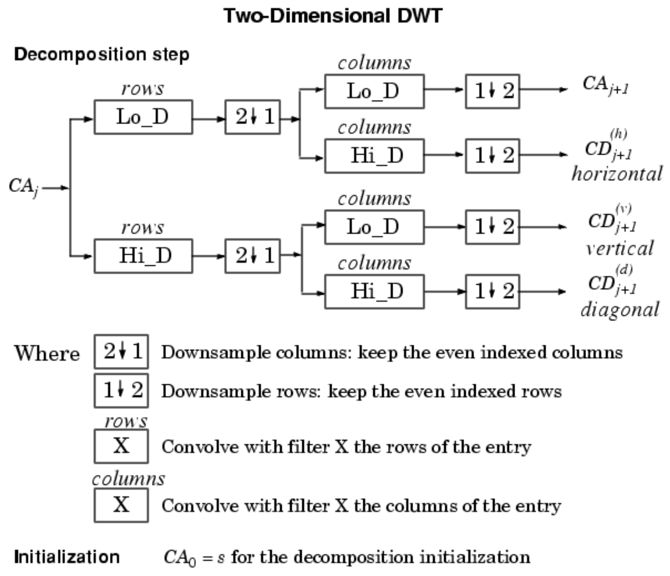


FIG1

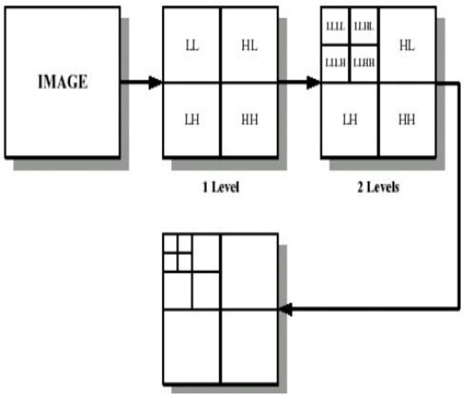


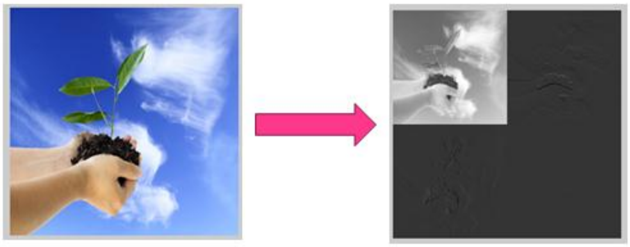
FIG2

III. DESCRIPTION

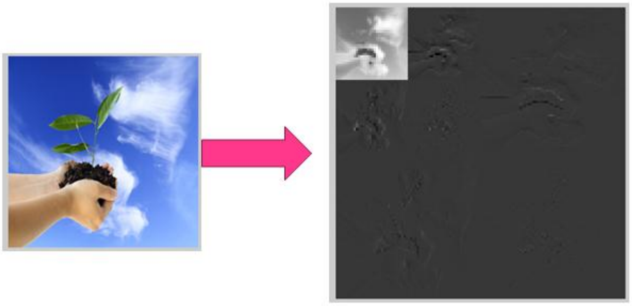
"Discrete Wavelet Transform", transforms discrete signal from time domain into time-frequency domain. The transformation product is set of coefficients organized in the way that enables not only spectrum analyses of the signal, but also spectral behavior of the signal in time .This is achieved by decomposing signal, breaking it into two components, each caring information about source signal. Filters from the filter bank used for decomposition come in pairs: low pass and high pass. The filtering is succeeded by down sampling (obtained filtering result Is "re-sampled" so that every second coefficient is kept).Low pass filtered signal contains information about slow changing component of the signal, looking very similar to the original signal, only two times shorter in term of number of samples. High pass filtered signal contains information about fast changing component of the signal. In most cases high pass component is not so rich with data offering good property for compression. In some cases ,such as audio or video signal, it is possible to discard some of the samples of the high pass component without noticing any significant changes in signal. Filters from the filter bank are called "wavelets".

IV.IMPLEMENTATION

1-level decomposition of original image



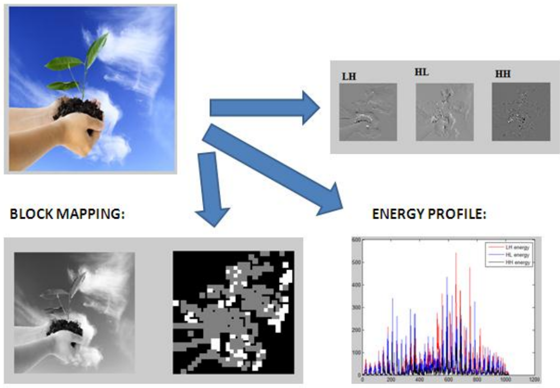
3-level decomposition of original image



A. *Dictionary Training*

Dictionary training is a much more recent approach to dictionary design, and as such, has been strongly influenced by the latest advances in sparse representation theory and algorithms. The most recent training methods focus on and sparsity measures, which lead to simple formulations and enable the use of recently developed efficient sparse coding techniques The main advantage of trained dictionaries is that they lead to state-of-the-art results in many practical signal processing applications. The cost Vas in the case of the KLT is a dictionary with no known inner structure or fast implementation. Thus, the most recent contributions to the field employ parametric models in the training process, which produce structured dictionaries, and offer several advantages. A different development, which we do not discuss here, is the recent advancement in online dictionary learning which allows training dictionaries from very large sets of examples, and is found to accelerate convergence and improve the trained result.

4.1 **Results So Far For Dictionary Training:**



IV. CONCLUSION

In this paper, we have briefly reviewed sparse and redundant representations as a new model that harnesses the local low-dimensional structure of natural images. Dictionary learning is a central step in employing a sparsity based model for various data processing tasks. Therefore, the speed of such learning algorithms is key in making many algorithms more efficient and thus more practical. In this paper we have also propose two simple yet effective modifications for the KSVD learning algorithm and an image watermarking technique based on a 3-level discrete wavelet transform has been implemented.

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