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MATHEMATICAL MODEL FOR INVENTORY CONTROL PROBLEM USING IMPRECISE PARAMETERS

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Abstract. In this paper, an inventory control problem is discussed using imprecise parameters. The fusion of geometric programming and fuzzy logic is used as imprecise parameters to solve inventory control problems. In inventory, holding costs, set-up costs, etc. may be flexible due to vague information. Fuzzy set theory is used to convert the inventory model crisp to fuzzy for producing flexible output. Compensatory operator is used to aggregate the fuzzy membership functions corresponding to fuzzy sets for fuzzy objectives and constraints. This aggregation gives the overall achievement function and the model known as fuzzy geometric programming model.

Keyword. Fuzzy objective; Fuzzy constraint; Compensatory operator; Achievement function; Geometric programming.

INTRODUCTION

Geometric Programming [1, 2] provides a remarkable optimization technique for solving a wide class of design and decision problems such as marketing mix problems, inventory control problems, personnel assignment problems, etc. These problems often found in practice, usually involve either posynomial or some posynomial objectives and/or constraints. Several authors [3, 4] have made valuable contributions to advance this filed. In any industry and any business, through the inventories are essential but this means lock up of capital. The excess inventories are undesirable which calls for controlling the inventories in the most profitable way. The different types of costs (ordering cost, carrying cost, understanding cost, over stocking cost, etc.) involved in the inventory problems to effect the efficiency of an inventory systems starts with the determination or economic order quantity. In inventory, holding costs, set-up costs, purchase price or product costs etc. may be flexible with vagueness in their values. All these parameters are normally variable and imprecise. Due to imprecise nature of the parameters, the problem becomes fuzzy and unsolvable. The concept of fuzzy set theory [5] can be applied to solve the problem. It produces the mathematical model [6] of imprecise information. Many authors [7-14] have developed different models of decision making problems in fuzzy environment. In this paper, using different achievement functions of fuzzy geometric programming inventory control problem is solved.

PRELIMINARIES

In this paper, some basic components of artificial intelligence and soft computing are used to enhance the performance of the proposed method. Artificial intelligence is a technique that able to perform tasks normally requiring human intelligence, such as visual perception, speech recognition, decision-making, and translation between languages [15]. Soft computing is a sub set of artificial intelligence which is a fusion of some techniques such as fuzzy logic, genetic algorithm, artificial neural network and inherent elements of them [16-17]. Fuzzy logic is a multi-value logic which is used to assign membership value in each element of the variable. This membership function is depending on the degree of membership function and nature of the variable. The nature of the variable is linguistic which can be expressed as low, medium, high, very high, etc. In last decade, fuzzy logic is used in several applications such as wireless sensor network [18], wireless ad-hoc network [19-21], inventory control problem, e-commerce [22], and other several areas. Geometric programming is a part of artificial intelligence which is used for non-linear optimization. It works with posynomial environment where nature of objective or constraint is posynomial.

MATHEMATICAL FORMULATION

The formulation of the inventory problem the following assumptions are given:

i. There are k products $p_1, p_2, \ldots, p_k$

ii. The annual demand is C
iii. The economic lot sizes be x_1, x_2, ……., x_k
iv. The set up costs including the production costs per run be c_1, c_2, …..c_k
v. h_1, h_2, ….., h_k be the storage costs per unit of the product per unit time
vi. i_1, i_2, ……, i_k be the number of unit that can be stored in a time interval
vii. a_1, a_2, ……, a_k are the advertising cost per unit of product

Since d_1, d_2, ……, d_k are the annual demands of the product, x_1, x_2, …x_k are the economic lot sizes, the number of runs per annum to fulfil their annual demands are $\frac{d_1}{x_1}$, $\frac{d_2}{x_2}$, ... ,$\frac{d_k}{x_k}$, respectively. If c_1, c_2, ...c_k are the production cost including the setup cost per run of the k products. Therefore, their production costs are $\frac{c_1 d_1}{x_1}$, $\frac{c_2 d_2}{x_2}$, ... ,$\frac{c_k d_k}{x_k}$. The total product cost is shown in Eq. 1

$$f_1(x)=\frac{c_1 d_1}{x_1}+\frac{c_2 d_2}{x_2}+\ \ldots\ldots+\frac{c_k d_k}{x_k} \qquad \ldots (1)$$

Since x_1, x_2, ... ,x_k are the economic lot size of the products p_1, p_2, ... p_k, respectively, and their average stock per unit time are $\frac{x_1}{2}$, $\frac{x_2}{2}$, ...$\frac{x_k}{2}$, respectively. If h_1, h_2, ... , h_k are the holding or storage cost then the total holding cost shown in Eq. 2.

$$f_2(x)=\frac{h_1 x_1}{2}+\frac{h_2 x_2}{2}+\ \ldots\ldots+\frac{h_k x_k}{2} \qquad \ldots (2)$$

Since, i_1, i_2, ... i_k are indicate the limitations of spaces, then the ranges economic of lot sizes are defined as:

$x_1 \le i_1$, or $\frac{x_1}{i_1} \le 1$,

$x_2 \le i_2$, or $\frac{x_2}{i_2} \le 1$,

…

…

$x_k \le i_k$, or $\frac{x_k}{i_k} \le 1$.

Minimize $f(x) = f_1(x) + f_2(x)$

Subject to $g_1(\underline{x}) = \frac{x_1}{i_1} \le 1$

$g_2(\underline{x}) = \frac{x_2}{i_2} \le 1$ … (3)

…

….

$g_k(\underline{x}) = \frac{x_k}{i_k} \le 1$

$x_1 \ge 0,\ x_2 \ge 0,\ \ldots\ldots\ x_k \ge 0$

Obviously, Eq. 3 is a Geometric Programming problem, in particular if limitations in the number of unit of all the product be L, then advertising cost is A which is shown in Eq. 4.

$$a_1x_1+,\ldots\ldots\ldots+a_kx_k \qquad \ldots (4)$$

The problem can be written as in Eq. 5 given as:

Minimize $(f(x), g(x))$

Subject to $h(x)=x_1, x_2+,\ldots\ldots+x_k \le L$ … (5)

$\frac{x_1}{L}+\frac{x_2}{L}+,\ldots\ldots\ldots+\frac{x_k}{L} \le 1$

$x_1 \ge 0,\ x_2 \ge 0, \ldots\ldots x_k \ge 0$

The goal version of Eq. 5 may be stated in Eq. 6.

Find $\underline{x} = (x_1, x_2, \ldots\ldots\ldots\ldots x_k)$

Subject to $f(\underline{x}) \le C$

$g(\underline{x}) \le A$ … (6)

$h(x) \leq L$

$x \geq 0$

Where x is the economic lot size vector. As stated earlier Eq. 6 is the goal programming model of Eq. 3. The fuzzified version of Eq. 3 can be stated as in Eq. 7.

$$\begin{array}{ll} \text{Find} & \underline{x} = (x_1, x_2, \ldots\ldots\ldots x_k) \\ \text{Subject to} & f(\underline{x}) \lessapprox C \\ & g(\underline{x}) \lessapprox A \\ & h(x) \lessapprox L \\ & x \geq 0 \end{array} \quad \ldots (7)$$

Where the wavy bar '~' stands for the word "approximately". Here C and A represents fuzzy aspiration levels of the costs and space, respectively. The problem in Eq. 7 is nothing but a fuzzy geometric programming problem.

If d_1, d_2, d_3 be allowable tolerances in the deviations from these fuzzy goals, respectively then the membership functions of the corresponding fuzzy sets can be stated as:

$$\mu_1 f(\underline{x}) = \frac{C-f(x)}{d_1}$$

$$\mu_2 f(\underline{x}) = \frac{L-f(x)}{d_2}$$

$$\mu_3 f(\underline{x}) = \frac{A-f(x)}{d_3}$$

We may use the set of achievement function $S=\{s_1, s_2\}$ where the values of s_1 and s_2 are defined as in Eqs. 8 & 9.

$$s_1 = \mu_1 f(x) + \mu_2 g(x) + \mu_3 h(x) \quad \ldots (8)$$

$$s_2 = (\mu_1 f(x))^2 + (\mu_2 g(x))^2 + (\mu_3 h(x))^2 \quad \ldots (9)$$

The compensatory model of the problem is given as in Eq. 10.

$$\begin{array}{ll} \text{Maximize} & s1 \\ \text{Subject to} & \mu_1 f(\underline{x}) = \frac{C-f(x)}{d_1} \\ & \mu_2 f(\underline{x}) = \frac{L-f(x)}{d_2} \\ & \mu_3 f(\underline{x}) = \frac{A-f(x)}{d_3} \\ & \mu_1 f(x) \geq 0,\ \mu_2 g(x) \geq 0,\ x \geq 0 \end{array} \quad \ldots (10)$$

The quadratic (canonical form) form of Eq. 10 is given in Eq. 11.

$$\begin{array}{ll} \text{Maximize} & s2 \\ \text{Subject to} & \mu_1 f(\underline{x}) = \frac{C-f(x)}{d_1} \\ & \mu_2 f(\underline{x}) = \frac{L-f(x)}{d_2} \\ & \mu_3 f(\underline{x}) = \frac{A-f(x)}{d_3} \\ & \mu_1 f(x) \geq 0,\ \mu_2 g(x) \geq 0,\ x \geq 0 \end{array} \quad \ldots (11)$$

SAMPLE PROBLEM

Consider the problem of minimizing the total inventory cost associated with ordering and holding of the inventory while satisfying certain warehouse space limitations. Suppose the annual demand for each product is 1000 units and the rate of demand is 0.2 per unit time. Thus, if the order size of product is x_1, units and the stock is depleted at a constant rate 0.2, then the stock of **Product 1** will be 0 after $\frac{x_1}{0.2}$ units of time. Similarly, if x_2 is the lot size of **Product 1** this stock will be 0 after $\frac{x_2}{0.2}$ units of time. We assume lot size orders of size x_1 and x_2, respectively to arrive, simultaneously.

Since, 1000 is the annual demand of **Product 1**, $\frac{1000}{x_1}$ is approximately the number of orders of **Product 1** per year. With 10 as the cost of a single order of **Product 1**, then 10 X $\frac{1000}{x_1} = \frac{10,000}{x_1}$, is the annual cost ordering of **Product 1**.

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Similarly, $\frac{1000}{x_2}$ is approximately the number of orders of **Product 2** per year. If 12 is the cost of a single order or **Product 2**, then 12 X $\frac{10,000}{x_2}=\frac{12,000}{x_2}$ is the annual ordering cost of product 12. Thus, the total ordering cost is $\frac{10,000}{x_1}+\frac{12,000}{x_2}$.

Since, the inventories are depleted at a constant rate, the average inventories are $\frac{x_1}{2}$and $\frac{x_2}{2}$. Unit storage costs are Rs. 5 and Rs. 4 for product 1 and 2 respectively. The average storage costs are 5 X $\frac{x_1}{2}$ and 4 X $\frac{x_2}{2}$. Therefore, inventory cost is given in Eq. 12.

$$f(x)=(2.5^*x_1 + \frac{10,000}{x_1} + 2^*x_1 + \frac{12,000}{x_2}) \qquad \ldots (12)$$

While lot sizes must be chosen to minimize f(x), the storage space requirements of x_1 and x_2 must not exceed 300 units, advertising cost per unit of x_1 product is 2 rupees, and x_2 product is 3 rupees. Thurs, total advertising cost must be satisfy $2x_1 + 3x_2 <= 600$.

Therefore, GP model of the problem given as in Eq. 13.

$$\begin{aligned}
&\text{Min} \quad f(x)=5x_1+\frac{10,000}{x_1}+2x_2+\frac{12,000}{x_2}\\
&\text{Subject to } g(x)=\frac{x_1}{300} + \frac{x_2}{300} \leq 1 \qquad \ldots (13)\\
&\quad h(x)=2x1 + 3x2 \leq 600\\
&\quad x1\geq 0,\ x2\geq 0
\end{aligned}$$

The solution is f(x)=757.0522632, x1=44.72136, x2=77.45967.

The problem can be written as FGP shown in Eq. 14.

$$\begin{aligned}
&\text{Determinatex}\\
&\text{Subject to} \quad f(x) \lesssim 50 \qquad \ldots (14)\\
&\quad g(x) \lesssim 1\\
&\quad h(x) \lesssim 600\\
&\quad x\geq 0
\end{aligned}$$

The problem of Eq. (14) is defuzzified as followed by linear membership function corresponding to the three fuzzy goals that are defined as:

$$\mu_1 f(x)=\frac{(800-f(x))}{t_1}$$

$$\mu_2 f(x)=\frac{(1.2-g(x))}{t_2}$$

$$\mu_3 f(x)=\frac{(650-h(x))}{t_3}$$

The compensatory model of FGP with equivalent objectives is given as:

Max s_1

$$\mu_1 f(x)=\frac{(800-f(x))}{t_1}$$

$$\mu_2 f(x)=\frac{(1.2-g(x))}{t_2}$$

$$\mu_3 f(x)=\frac{(650-h(x))}{t_3}$$

$\mu_1 f(x)\geq 0$, $\mu_2 g(x) \geq 0$, $\mu_3 h(x) \geq 0$, $\mu_1 f(x)\leq 1$, $\mu_2 g(x) \leq 1$, $\mu_3 h(x) \leq 1$, $x_1\geq 0$, $x_2\geq 0$, $x_3\geq 0$;

The canonical model of FGP with equivalent objectives is given as:

Max s_2

$\mu_1 f(x)=\frac{(800-f(x))}{t_1}$

$\mu_2 f(x)=\frac{(1.2-g(x))}{t_2}$

$\mu_3 f(x)=\frac{(650-h(x))}{t_3}$

$\mu_1 f(x)\geq 0$, $\mu_2 g(x)\geq 0$, $\mu_3 h(x)\geq 0$, $\mu_1 f(x)\leq 1$, $\mu_2 g(x)\leq 1$, $\mu_3 h(x)\leq 1$, $x_1\geq 0$, $x_2\geq 0$, $x_3\geq 0$;

The performance evaluation of both solutions is shown in Table 1.

Table 1: Performance evaluation of both solutions

Exp. No.	t_1	t_2	t_3	s_1		s_2	
				Decision Variables	Objective Value	Decision Variables	Objective Value
1	20	0.5	40	x_1=50.00021 x_2=128.1969	800.0000176	x_1=50.00021 x_2=128.1969	800.0000176
2	30	.75	50	x_1=48.63632 x_2=59.12437	769.9999946	x_1=48.63632 x_2=59.12437	769.9999946
3	40	.10	45	x_1=48.85902 x_2=70.94684	760.0000046	x_1=48.85902 x_2=70.94684	760.0000046
4	50	.15	48	x_1=44.72136 x_2=77.45967	757.0522632	x_1=31.69364 x_2=77.94990	783.8337504

CONCLUSION

With the introduction of fuzzy set applications to many areas of real life decision making problems, the art of decision making has become eloquent and realistic. It is observed that it can also be applied to a class of inventory control problem where parameters are neither precisely defined nor precisely measured. These complexities make the use of the theory of fuzzy more and more attractively. The proposed model can be applied in multi item inventory control problems under many other limitations such as their inventory levels, warehouse and various other contractual constraints.

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