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SOLUTION AND PERFORMANCE EVALUATION OF TRANS-SHIPMENT PROBLEM USING A MINIMUM SPANNING TREE APPROACH

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Abstract

Purpose: Transportation problem plays an important role in operations research. The more generalized cases of transportation problems are trans-shipment problems. Further, the trans-shipment problems may have a set of trans-shipment nodes, or the source/destination nodes themselves act as the trans-shipment nodes. The study of the trans-shipment problems and their solution methodology is the goal of this paper.

Methodology: The solution of a trans-shipment problem could be done by transferring it to a transportation problem. Further, there exist various conventional methods for solving the transportation problem. The present paper discusses about the scope of application of an existing heuristic algorithm directly over the trans-shipment problem. The heuristic is based on the minimum spanning tree approach. We implement the algorithm over a test problem and further compare its performance by the performance of the corresponding algorithm Vogel's Approximation Method.

Main findings: The spanning tree approach gives a better solution or almost the nearby solution as compared to the solution obtained by Vogel's Approximation Method.

Implications: The solution obtained by the spanning-tree approach takes lesser computational effort to reach a better feasible solution.

The novelty of study: The algorithm to deal with the trans-shipment problem i.e. for finding the feasible solution of the trans-shipment problem is the main focus of this paper.

Keywords: *Transportation Problem, Trans-shipment Problem, Vogel's Approximation Method (VAM), Kruskal's Algorithm, Minimum Spanning Tree.*

INTRODUCTION

The transportation problem in the literature and under the discussion for this paper may be defined as follows:

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

$$\text{Subject to} \sum_{i=1}^{m} x_{ij} \le b_j ,\ j = 1,2\ldots n ,\ \sum_{j=1}^{n} x_{ij} \le a_i , i = 1,2,\ldots m ,\ x_{ij} \ge 0$$

Where x_{ij} is the amount to be shipped from source i to destination j. a_i is the supply available at the source i and b_j is the demand at the destination j. The above model is called the linear model of a given transportation problem with m sources and n destination.

Again, the trans-shipment problem under consideration for this paper is defined as follows Agadaga, G. O. & Akpan, N. P. (2017)):

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \tag{2}$$

$$\text{Subject to } \sum_{arcout} x_{ij} - \sum_{arcin} x_{ij} = a_i \quad \text{(constraints for the source node } i\text{)}$$

$$\sum_{arcout} x_{ij} - \sum_{arcin} x_{ij} = 0 \quad \text{(constraints for the trans-shipment node)}$$

$$\sum_{arcout} x_{ij} - \sum_{arcin} x_{ij} = b_j \quad \text{(constraints for the destination node } j\text{)}$$

$x_{ij} \geq 0$.

In a similar manner to the transportation model, x_{ij} is the amount to be shipped from source i to destination j and it's a decision variable. a_i Is the supply available at the source i and b_j is the demand at the destination j. The additional thing which is coming in the picture is trans-shipment constraints, which creates a difference from the transportation problem. The next paragraph will describe the difference between transportation and trans-shipment problems more precisely.

A transportation problem is a problem with some given sources and destinations and with given availability of resources and demand at the respective sources and destinations. A trans-shipment problem has trans-shipment nodes in addition to all these sources and destinations. A trans-shipment node is defined as the mediator node between sources and destinations. The number of trans-shipment nodes may be single or more than one and may vary for different types of problems. Sometimes the trans-shipment nodes are absent and source/destination nodes act as trans-shipment nodes. The following four pictures define and distinguish clearly transportation and transshipment problems.

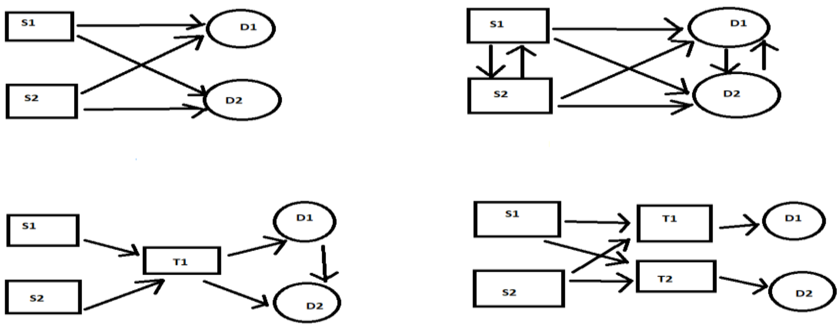

Figure 1(a), 1(b), 1(c) and 1(d): Different types of transportation/trans-shipment problems

Consider the pictorial representations of four different situations. Figure 1(a) represents a transportation problem while Figure 1(b), 1(c), and 1(d) represent different types of trans-shipment problems. In terms of graph theory, Figure 1(a) is a bipartite graph and others are not. We have no distinct trans-shipment node in Figure 1(b), the source and destinations are acting as the trans-shipment nodes there. In Figure 1(c) and 1(d), there are distinct trans-shipment nodes. When there are distinct trans-shipment nodes, the source/destination nodes may or may not work as the trans-shipment nodes. For example, see Figure 1(c) where the destination node D1 is working as a trans-shipment node. In Figure 1(d), no source/destination nodes are working as the trans-shipment node.

In the present paper, we are trying to solve the trans-shipment problems which are similar/equivalent to the structure of Figure 1(b) only, i.e. from next time if we use the word 'trans-shipment problems', it resembles only the trans-shipment problems of structure similar to Figure 1(b). The general way for solving a trans-shipment problem is to convert it into a transportation problem first, and then the solution is obtained by any of North-West corner rule, matrix minima, or Vogel's approximation method. The solutions obtained by these conventional methods are not necessarily optimal. We apply a modified distribution method or u-v method for checking the optimality of these feasible solutions.

We are applying a different strategy for directly dealing with the given trans-shipment problem (Agadaga, G. O. & Akpan, N. P. (2017)). The method is based on a minimum spanning tree. Before going into detail, we would like to discuss briefly the existing literature in this area. Earlier in 1999, Multi objective-based transportation problem was solved in Gen, M. et al., (1999), by spanning tree-based genetic algorithm. The solution of transportation problems was done in Aljanabi, K. B. & Jasim, A. N. (2015), by using an entirely new idea, i. e. by using the minimum spanning tree method. They used a modified Kruskal method for dealing with spanning trees. Again, a similar method was used in Akpan, N. P. & Iwok, I. A. (2017), for dealing with a different transportation problem. It obtained the solution graphically and declared by comparing that the obtained solution is optimal also. Optimal ship transportation is done in (Antoš, K. (2016)) by using a similar kind of approach of minimum spanning tree. Nonlinear fixed charge transportation was solved by spanning tree-based genetic algorithm (Jo, J. et al., (2007)). Step fixed size transportation problem is solved in (Molla, A. Z. S. et al., (2014)) by spanning tree-based memetic algorithm. Since the field of dealing with a transportation problem with a spanning tree is new, lesser work has been done till now.

In this paper, we are using a similar strategy used by Aljanabi, K. B. & Jasim, A. N. (2015), to obtain the solution of the trans-shipment problem. The solution obtained in this way is feasible. We then compare the solution obtained by this method to the one obtained by the conventional method Vogel's approximation method. The entire work is justified by a successful implementation of the algorithm over a test problem. (Dubey, O. P., Deep, K. & Nagar, *A. (*2014))

METHODOLOGY

Before going into the details of the methodology of the present work, we'd like to introduce the Kruskal method (Greenberg, H. J. (1998), Kruskal, J. B., (1956)) for spanning tree/shortest path problem. Suppose we have a graph consisting of N nodes and E edges with each edge assigned a given cost. Then, the following algorithm gives a minimum spanning tree.

Kruskal (N, E, cost):
Sort the given edges in E by increasing cost
While |T|<|N|-1
Let (u,v) be the next edge in E in terms of the lowest cost
If u and v are belonging from the different components, join the components of u and v
T=T ∪ {(u,v)}
Return T

Conversion of transshipment problem to spanning tree

If we have a given trans-shipment problem similar to that of figure 1(b), we can easily convert that in a spanning tree problem. We'd like to give an example of the same.

Consider the following trans-shipment problem:

Table 1: A simple example of a transshipment problem

From\To	S1	S2	D1	D2	Supply
S1	M	2	2	5	200
S2	4	M	4	1	600
D1	0	0	M	6	
D2	0	0	4	M	
Demand			400	400	

The above example consists of two sources and two destinations only. The cost of transportation from each node to a different node is given. Since we can't supply anything from a source to itself, the transportation cost is a big number M. The spanning-tree conversion (similar to that in (Akpan, N. P. & Iwok, I. A. (2017), Aljanabi, K. B. & Jasim, A. N. (2015)) to the above problem looks like Figure 2.

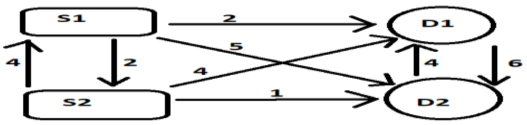

Figure 2: Graphical form of the trans-shipment problem

In the above spanning tree, we are considering the cost of each arc as the cost given in the trans-shipment table. The above graph problem is solvable easily as a minimum spanning tree problem.

For solving the above type of graph problem with directed arc, we could apply our algorithm which is inspired from Kruskal algorithm (Akpan, N. P. & Iwok, I. A. (2017), Aljanabi, K. B. & Jasim, A. N. (2015) & Kruskal, J. B., (1956)) and is useful for finding the feasible solution only.

PROPOSED ALGORITHM

Following are the simple steps of the algorithm that we to follow for finding the feasible solution of the above type of trans-shipment problem (after converting it to the graph problem):

- Consider all the destination nodes first, randomly pick a node and choose the one arc of minimum cost amongst all the arcs connected with that node to the source nodes.
- Supply maximum possible amount from the source from the chosen arc.
- Repeat the process until all the destination nodes are connected to some source node.
- The remaining source node(s) (if any) is/are to be connected with any other source /destination node by searching again an arc with minimum cost.
- Supply the maximum amount from the connected arc.
- Repeat the process until all source nodes are connected to at least one other source/destination node.

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We'd like to illustrate the above algorithm by running over a more tedious problem with four sources and two destinations.

Consider the following trans-shipment problem:

Table 2: An illustration of trans-shipment problem with four sources and two demand

From\To	S1	S2	S3	S4	D1	D2	Supply
S1	M	5	24	6	24	10	100
S2	8	M	6	10	5	20	150
S3	45	20	M	7	45	8	200
S4	20	25	10	M	30	7	350
D1	15	20	62	15	M	8	
D2	8	25	25	24	5	M	
Demand					400	400	800

The step by step procedure for solving the above problem is represented below with the help of the following diagrams:

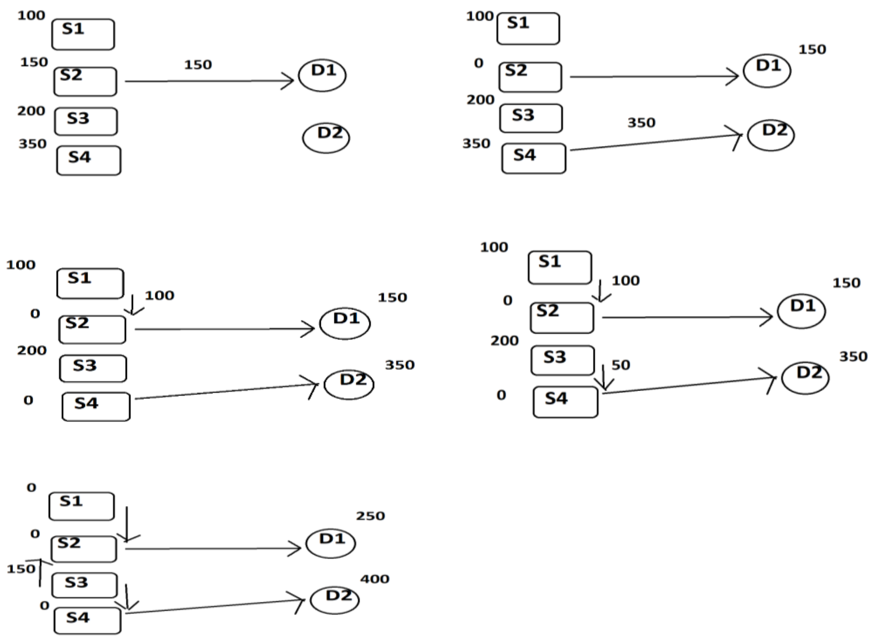

Figure 3(a), 3(b), 3(c), 3(d) and 3(e): Step by step solution of the trans-shipment problem by a new heuristic algorithm

The above arrow diagrams show the various stages we followed during the implementation of the above-proposed algorithm. The solution obtained by the above implementation is shown in the table below:

Table 3: solution of the illustrated trans-shipment problem by a new heuristic algorithm

From\To	S1	S2	S3	S4	D1	D2	Supply
S1	M	5 (100)	24	6	24	10	100
S2	8	M	6	10	5 (150+100+150)	20	150
S3	45	20 (150)	M	7 (50)	45	8	200
S4	20	25	10	M	30	7 (350+50)	350
D1	15	20	62	15	M	8	
D2	8	25	25	24	5	M	
Demand					400	400	800

The solution or the total cost obtained from the algorithm is therefore given by $5\times100+5\times400+20\times150+7\times50+7\times400=8650$. Although the solution obtained above is obtained from selecting the minimum cost arcs during all the steps of the algorithm, the solution obtained is not optimal. However, it is closer to the optimal solution. The optimality of the solution could easily be checked by using the famous method called the modified distribution method. In our present paper, we have not checked the optimality of the obtained solution; rather we have compared the obtained solution with an existing method, Vogel's approximation method.

Since Vogel's approximation method is a well-known method, we are not writing the details of the solution, instead, we are putting the final solution of the above problem directly. The following table gives the required solution:

Table 4: Solution of the illustrated trans-shipment problem by Vogel's approximation method

From\To	S1	S2	S3	S4	D1	D2	
S1	0	5 (100)	24	6	24	10	100
S2	8	0	6	10	5 (150+100+150)	20	150
S3	45	20 (150)	0	7	45	8(50)	200
S4	20	25	10	0	30	7 (350)	350
D1	15	20	62	15	0	8	
D2	8	25	25	24	5	0	
					400	400	800

The obtained solution through Vogel's approximation is given by 5×100+5×400+20×150+8×50+7×350=8350. For the present example, our result is closer to the result of Vogel's approximation method.

CONCLUSION AND FUTURE SCOPE

In many cases, the results of the proposed minimum spanning tree method and Vogel's approximation method are equal or comparable/closer. Therefore we may term this algorithm as a heuristic approach for trans-shipment problems. One most important benefit of this algorithm is that we get the feasible solution is comparatively lower computational effort, which is the only point we want to infer from this paper. The optimal solution is not guaranteed, however, it could be obtained in a conventional way which is a modified distribution method. The scope of this heuristic approach could further be probed to all other types of trans-shipment problem. We can also find some of the hybrid heuristic/methods by mixing this strategy with other existing methods.

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