Transportation problem with restriction on arcs and their Solution by mathematical modeling

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Abstract

Purpose of the study: Transportation problem plays an important role in logistics. The present paper introduces the transportation problem along with some of its variants and their mathematical modeling.

Methodology: The mathematical modeling is done for restricted path transportation problem. We have dealt with the transportation problems with specified number of blocked arcs or arcs with limited allowance through them. We have applied the same on some test problems chosen arbitrarily. The application is also extended on the transportation problems with some additional source-destination related restrictions.

Main Findings: The optimal cost of constrained transportation problems under the study is obtained.

Applications of this study: The process could be integrated on a larger scale in logistics.

Novelty/Originality of this study: The study is done for special types of transportation problems having specific number of blocked arcs or arcs with limited allowance through them. Some more source-destination related constrained are also dealt in this paper.

INTRODUCTION

Operations research consists of a lot of problems. For example, assignment problem transportation problem, traveling salesman problem, knapsack problem etc. We are going to study the transportation problem in the next couple of sections. It is the most basic problem of operations research. Before we proceed, let's try to understand the problem first.

Suppose we have some given number of sources and some given number of destinations. They may be more than one. Now, we have to transport some goods or quantities from sources to destinations. The cost of transportation through each arc is already given. We need to meet all the transportation from the given sources to destination and at the same time we have to minimize the transportation cost also. So, the optimal allocation is to be made in such a way that we must minimize the cost of transportation. We have to take care of the number of availability at sources as well as the requirement at the destination points.

Transportation problem has a lot of application in our day to day life. Starting from a pizza delivery to big ship transportation, there are a lot of applications. So, this is not only a conventional area but also an evergreen area of operations research. We need to do research and develop a lot of techniques to deal with the day to day challenging problems arising in logistics and transportation.

In the present paper, we are going to deal with the transportation problems consisting of restriction on some arcs and some more source-destination related conditions. The ideas presented are inspired from some of the existing literatures (Khurana, A. (2015), Proll, L.G., (1973), Gupta, K. and Arora, S.R. (2013), and Gupta, K. and Arora, S.R. (2012). The originality of the present work includes the randomly chosen examples along with their solutions through mathematical modeling.

Before proceeding ahead, we'd like to have a look at some of the existing literatures in this field. A lot of literature exists in this field. Variants of transshipment problems are introduced by Khurana, A. (2015). A transshipment problem is simply an extension of transportation problem. In his paper, he has introduced some complex variants of transshipment problems. A similar kind of study regarding variants of transportation is found in (Proll, L.G., (1973)). Fractional capacitated transportation problem with restricted flow is studied by Gupta and Arora in Gupta, K. and Arora, S.R. (2013), Gupta, K. and Arora, S.R. (2012). The restriction on arc idea is found in this paper. The two stage time minimization with restricted flow is studied by Kaur, P., Verma, V., and Dahiya, K. (2017). This paper has also used the concept of restricted flow. Singh, P. and Saxena, P.K. (1998) studied about shipping cost/completion-date trade-off in transportation problem with some additional restriction. The resource restricted problem is studied in Moon, I.K. and Cha, B.C. (2006), which is written for production-inventory system. The transportation problem with multiple objectives is studied in Lee, S.M. and Moore, L.J. (1973). A multiple conflicting goals like various environmental constraints, unique organizational value of the firm

Although there are a vast number of literatures on transportation and transshipment with their variants, we study a simple model consisting a transportation problem with a complete restriction on some given arcs or the transportation is permitted partially from those given arcs. In addition to that, we also study the problem with some additional source-destination related restrictions. In the present paper, we study them with their mathematical models only.

**THE CONVENTIONAL TRANSPORTATION PROBLEM**

In the conventional transportation problem, we have m sources and n destinations. The availability at each sources are $a_i$, $i=1,2,...,m$ and the demand at destinations are $b_j$, $j=1,2,...,n$. If a balanced transportation problem is considered, we have the sum of availability at sources equals to the sum of demand at destinations. We have $c_{ij}$ the cost of transportation on each arc. The goal of a transportation problem is to find the minimum cost for meeting the demand at all destinations.

![Figure 1: A simple transportation problem with four sources and four destination points](https://mgesjournals.com/ijsrtm/)

Before proceeding further, we need to have an idea of the mathematical model of a given transportation problem. The following is a mathematical model of a balanced transportation problem with m sources and n destinations, which we are willing to consider in this paper.

Minimize

$$
\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}
$$

Subject to

$$
\sum_{j=1}^{n} x_{ij} = a_i, i=1,2,3,4,...,m
$$

$$
\sum_{i=1}^{m} x_{ij} = b_j, \quad j=1,2,3,4,...,n
$$

$$
x_{ij} \geq 0, \quad i=1,2,...,m; j=1,2,...,n
$$

$$
x_{ij} \in \mathbb{Z}, \quad i=1,2,...,m; j=1,2,...,n
$$

$$
x_{ij} \geq 0, \quad i=1,2,...,m; j=1,2,...,n
$$
Let’s try to define the transportation problem we are going to deal with. We’re going to deal with the transportation problem with some restriction over some arcs. That means either some arcs are fully restricted or some arcs are allowed with a given amount of transportation only. That can happen in general i.e. in real life scenario the transportation path may get locked due to some reason or a mild allowance is there on some particular path.

In the present paper, we are simply applying an independent idea which is inspired from Khurana, A. (2015). We are applying the same over some independent transportation problems through LINDO software.

The following is a possible mathematical model of a balanced restricted path transportation problem.

Minimize
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]  \hspace{1cm} (2)

Subject to
\[
\sum_{j=1}^{n} x_{ij} = a_i, \, i = 1,2,3,4, \ldots m
\]
\[
\sum_{i=1}^{m} x_{ij} = b_j, \, j = 1,2,3,4, \ldots n
\]
\[
x_{ij} \geq 0, \, \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \leq d_{ij}
\]

Here, \(d_{ij}\) represents the permissible amount of transportation through source node \(i\) and destination node \(j\). Due to this, a single additional constraint coming in the problem and making the entire problem unique.

Consider an example of two sources and two destinations to understand the situation under consideration. The picture 2 depicts the situation. The path from source 2 to destination 2 is restricted i.e. we can’t supply more than \(d_{22}\) units from that path.

![Figure 2: An example with restricted arc transportation](image)

The above could be extended further to a problem with some specified number of blocked path also. In such cases, the last constraint is to be modified as follows:

Minimize
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]  \hspace{1cm} (3)

Subject to
\[
\sum_{j=1}^{n} x_{ij} = a_i, \, i = 1,2,3,4, \ldots m
\]

Further, we must remove the situation when all sources and all destination points are taken. Similarly, the destination nodes are to be adjusted later on. The same is to be followed for minimizing the overall cost.

\[ \sum_{i=1}^{m} x_{ij} = b_j, \ j = 1,2,3,4,\ldots,n \]
\[ x_{ij} \geq 0, \ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \]
\[ x_{ij} = 0 \] (for blocked paths only)

Further, the model could also be extended to the case with some source-destination related restriction. We are also willing to deal only those restrictions which are precisely defined in the next section.

**SOURCE-DESTINATION RELATED RESTRICTION**

We extend our model with additional source-destination related restrictions. The condition could be stated be as follows: A nonempty subset of destination node(s) is/are to be satisfied by a nonempty subset of source node(s) on priority basis and the remaining sources destinations are to be adjusted later on. The same is to be followed for minimizing the overall cost involved. Understanding this condition will be easier if we focus on the previous example. If we look at the example and impose an additional constraint that the demand at destination node \( j = 1, 2 \) only are to be done first by the source node \( i = 1 \) only. That means a subset of destinations is/are to be satisfied by a subset of source node(s) first and the remaining demands at destination(s) (which may or may not include subset destination node(s)) is/are to be adjusted by the source(s) (which may or may not include subset source node(s)) and ultimately minimizing the overall cost of transportation. We can impose multiple such restrictions in one model.

We’d like to generalize this source-destination related constraint now. Consider the transportation problem (1). The additional constraint imposed here could be stated as follows: Let \( S_i \) be the nonempty distinct subsets consisting elements taken from \( S_j \), i.e. \( S_k \subset [ \bigcup_{i} S_i, i = 1,2,3,4,\ldots,m ] \). Similarly let \( D_k \) be the nonempty distinct subsets taken as \( D_k \subset [ \bigcup_{j} D_j, j = 1,2,3,4,\ldots,n ] \). The subset constraints are imposed as given in (4). Also, the only one constraint in which \( S_k \) and \( D_k \) are using all the sources and destinations nodes respectively is avoided.

The mathematical model corresponding to that will be as follows:

Minimize
\[ \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \] (4)

Subject to
\[ \sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,3,4,\ldots,m \]
\[ \sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,3,4,\ldots,n \]
\[ x_{ij} \geq 0, \ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \]
\[ \sum_{i \in S_k} \sum_{j \in D_k} x_{ij} = \theta_k, k = 1,2,3,4,\ldots,\eta . \]

Where \( \theta_k \) is minimum of \( \sum_{i \in S_k} a_i \) or \( \sum_{j \in D_k} b_j \) (for a given \( k \)).

Here, \( \eta \) is representing the number of such constraints only. There may be situations where the source subset may consist of just one source node or the destination subset may consist of just one destination node only. We only avoid the situation where both source and destination subsets consist of all the sources and destination nodes respectively. We say the additional constraint coming this way as source-destination constraint. The number of constraints having the structure may be equal to or more than one i.e., \( k \geq 1 \) in a given model.

The value of \( \eta \) i.e. the number of such possible constraints also matters a lot. The number of possible subsets of source nodes is \( 2^m - 1 \). Similarly, the destination nodes have \( 2^n - 1 \) subsets. Therefore total number of such source-demand constraints is \( (2^m - 1)(2^n - 1) \). Further, we must remove the situation when all sources and all destination points are taken

70 | Visit IJSRTM at https://mgesjournals.com/ijsrtm/
as subsets. Therefore, the required number of source-demand constraint is \((2^n - 1)(2^n - 1) - 1\). Therefore, a model may consist of number of such source-demand constraint ranging from \(k = 1\) to \(k = \eta = (2^n - 1)(2^n - 1) - 1\).

The above constraints could be explained with the help of an example. Consider a balanced transportation problem with 9 sources and 9 destinations. The availability at sources and demand at different destinations are given. The two additional constraints are to be imposed as follows:

**Constraint 1:** The sources S2 and S3 must fulfill the demand at D4, D5, and D6 at priority

**Constraint 2:** The sources S7 and S8 must fulfill the demand at D7 and D8 at priority

The remaining transportation must be adjusted as per the ways of dealing transportation problem with minimizing the overall cost. It is also given that the value of \(a_2 = 10\), \(a_3 = 5\), \(b_4 = 20\), \(b_5 = 25\), \(b_6 = 25\), \(a_7 = 10\), \(a_8 = 10\), \(b_7 = 5\) and \(b_8 = 5\) for this example. Therefore, the following mathematical constraints are to be added in the general mathematical model of this given transportation problem example.

\[
x_{24} + x_{25} + x_{26} + x_{34} + x_{35} + x_{36} = 15
\]

\[
x_{77} + x_{78} + x_{87} + x_{88} = 10
\]

The application area of such examples is also diverse. If we have some restriction in our transportation problem that a set of certain destination nodes are to be satisfied by a set of sources node first, then we can follow the above model.

**METHODODOLOGY**

The existing method in literature like North West Corner Rule (NWCR), Least Cost Method (LCM) or, Vogel’s Approximation Method (VAM) for finding the feasible solution works on balanced transportation problem, particularly when the path is not restricted. For applying them over restricted transportation problem, we need to update those methods. Similarly, the optimality could be obtained by Modified Distribution (MODI) method but that further needs research and an update in the existing method.

We are simply using the mathematical model and software LINDO to deal the above model. Any software which is capable of dealing an Linear Programming Problem (LPP) can solve the model. We have used LINDO because the coding is much easier. The optimal solution is directly found by using the software.

For example, consider the picture 2 with some terms taken as \(c_{ij} = 2\) for all \(i\) and \(j\), \(a_1 = 10\), \(a_2 = 15\), \(b_1 = 20\) \(b_2 = 5\) and \(d_{22} = 1\), the maximum allowance on the arc joining S2 to D2. The LINDO code for the same is given as follows:

MINIMIZE 2X11+2X12+2X21+2X22

SUBJECT TO

\[
X_{11} + X_{12} = 10
\]

\[
X_{21} + X_{22} = 15
\]

\[
X_{11} + X_{21} = 20
\]

\[
X_{12} + X_{22} = 5
\]

\[
X_{22} \leq 1
\]

END

INT X11

INT X12

INT X21

INT X22

Now, some source-destination restricted is to be imposed and the LINDO code for the same is to be studied.

For example, consider the picture 2 with same values stated above and an additional restriction is imposed as follows: Demand at D1 is to be satisfied completely by S1 first. The remaining supplies and demands are to be satisfied later on.

MINIMIZE 2X11+2X12+2X21+2X22

SUBJECT TO

\[
X_{11} + X_{12} = 10
\]

\[
X_{21} + X_{22} = 15
\]
We’d like to solve some examples on the basis of above. Consider the following restricted path problem. Consider the cost mentioned below to be in Rupees. The four sources and five destinations are having different supply and demand which is indicated in the table. The total transportation available units are 80 only.

**Table 1: A transportation problem under consideration**

<table>
<thead>
<tr>
<th>From  \</th>
<th>To</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>D5</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>S2</td>
<td>10</td>
<td>2</td>
<td>9</td>
<td>6</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>S3</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>S4</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>5</td>
<td>30</td>
<td>15</td>
<td>20</td>
<td>80</td>
</tr>
</tbody>
</table>

When the above is modeled in LINDO similar to the program shown above, solution is obtained. The solution of the above problem gives an optimal cost of Rs. 195 for given transportation. The individual transportations are listed as follows: \(x_{11}=5, x_{12}=5, x_{13}=10, x_{23}=20, x_{33}=20, x_{41}=5, x_{44}=15\).

Now, we apply a restriction that we S2 to D5, we can send only 5 units and S4 to D1 only 1 unit. So, instead of the solution above, we must get a new solution. The LINDO model for the above needs additional two constraints. When modeled, we have an optimal solution.

The restricted solution simply shows that the cost is increased to Rs. 240 and we can observe the updated transportation on different paths. \(x_{11}=8, x_{13}=12, x_{22}=5, x_{23}=10, x_{33}=5, x_{35}=15, x_{41}=2, x_{43}=13, x_{44}=5\).

We have arbitrarily chosen some more problems and applied the same solution strategies and found impressive results. If we are willing to work with some additional source-destination related restriction, we can have a slightly different modeling. Consider the problem of table 1 again. In addition to the previous restriction, we apply one more source-destination related restriction, which is sources 1 and 2 must satisfy the demand of destination 1 and 2 first, then only the remaining supplies or demands are adjusted after that (keeping the lesser cost in consideration). For fulfilling this criterion, we apply an additional constraint \(X11+X12+X13+X14 \leq 15\), because the combined demand at D1 and D2 is 15 only.

**Figure 3:** The solution of i) transportation problem and ii) restricted transportation problem under consideration
Figure 4: The solution of i) transportation problem with an additional source-destination restriction and ii) restricted transportation problem with an additional source-destination restriction under consideration

We have applied the above additional constraint to both the cases considered above, viz. case I, the original problem and case II, the restricted assignment problem. The solution obtained for the first one is \( x_{11} = 10, x_{12} = 5, x_{13} = 5, x_{23} = 20, x_{33} = 20, x_{43} = 5, x_{44} = 15 \). The same for the second one is given by \( x_{11} = 10, x_{13} = 10, x_{22} = 5, x_{24} = 10, x_{25} = 5, x_{33} = 5, x_{35} = 15, x_{43} = 15, x_{44} = 5 \).

We’d like to tabulate the different transportation problem under consideration and the results found by using the LINDO software.

<table>
<thead>
<tr>
<th>Transportation problem example under consideration</th>
<th>General problem of table 1</th>
<th>General problem of table 1 with two restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source-destination restriction is not applicable</td>
<td>Optimal cost is Rs. 195</td>
<td>Optimal cost is Rs. 240</td>
</tr>
<tr>
<td>Source-destination restriction is applicable</td>
<td>Optimal cost is Rs. 195</td>
<td>Optimal cost is Rs. 240</td>
</tr>
</tbody>
</table>

CONCLUSION

The present paper could be a work for computationally checking the effect of some restriction over some arc on a given transportation problem. We have successfully dealt constrained path restrictions and additional source-destination restrictions. Mathematical model plays an important role in dealing various problems of operations research. If there is a variation in the problem, the mathematical model also gets updated. We can get the solution of the updated model by putting the changed mathematical model in the requisite software. The simple conclusion of this work is that a mathematical model is an efficient tool for any operations research model.

LIMITATION AND STUDY FORWARD

The present research explores the mathematical models and the solution obtained from that. The problem under consideration is constrained transportation problems only. The constraints are applied either on some arcs in terms of limitation on arcs or blocking some arcs. Also, sources-demand constraints are considered where subsets of sources and destination points are taken and should be satisfied on priority basis. Examples are taken to justify the idea.

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AUTHORS CONTRIBUTION

The authors confirm contribution to the manuscript as follows: study conception and design: RP, data analysis: RP; analysis and interpretation of results: RP; draft manuscript preparation: JP, OPD. All authors reviewed the results and JP and OPD approved the final version of the manuscript.

REFERENCES


