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A cost analysis on Multi-item Inventory model for Factory Outlets with investment constraint under ranking Asteroid Fuzzy Set

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Abstract

Purpose of the study: A multi-item inventory model for factory outlets in crisp and fuzzy sense are formulated in the fuzzy environment with investment under one constraint has been considered. In this model, demand is constant and is related to the price per unit item. The asteroid fuzzy set is defined and is properties are given.

Methodology: The parameters involved in this model represented by asteroid fuzzy set. The average total cost is defuzzify by ranking method.

Main Findings: The analytical expressions for maximum inventory level and average total cost are derived for the proposed model by using nonlinear programming technique. A numerical example is presented to illustrate the results.

Applications of this study: Ranking Asteroid fuzzy set is considered profitable in small businesses. Here we considered the factory outlet .Also its use is considered to help the small scale entrepreneurs during festival period and pandemic situation.

Novelty/Originality of this study: In this paper, a novel approach to handle the asteroid fuzzy set is proposed. It uses ranking the cost parameters of the asteroid fuzzy set with the best approximation level. The parameters involved are asteroid fuzzy set and are all ill-defined in nature.

INTRODUCTION

Cost parameters, objective functions, and decision makers' constraints are all imprecise in most real-world situations. The classical (EOQ) inventory problem is defined as the problem of determining the optimal order quantity under relatively stable conditions. This EOQ problem with varying variance had been solved for several years and published since 1915 by a number of researchers. Harries, F.W. (1990), Taft, E.W. (1918), and Hadley, G. & Whitin, T.M. (1958) discuss two major assumptions in the classical EOQ models: the demand rate is constant and deterministic. Uncertainties are treated as randomness in conversional inventory models, and they're dealt with using probability theory. However, in some cases, uncertainties are caused by fuzziness, and the fuzzy set theory can be used in these situations. The fuzzy inventory model with storage space and budget constraints was discussed by Chou, S., Peterson, C., Julian, B., Kuo-Chen, H. (2009). Moghdani, R., Sana, S.S., Zadeh, H.S.(2019) discussed a multi-item EPQ model that was developed for multi-packaging delivery by taking into account warehouse space constraints and the number of total orders, and the resulting fuzzy model was simplified using α-cuts and variable changes.

A factory outlet is outlined as a factory shop and it is a store where manufacturers sell their products directly to the public at steep discounts. Because not all of a company's products are of high quality, they cannot be sold in retail stores. Manufacturers have initiated to produce products specifically for outlet centres in order to avoid competing with their retail outlets, thanks to the industrial development of outlet centres. Adriana, F., Gabor, Jan-kees van Ommeren, Sleptchenko, A.(2011) discussed an inventory model for an Omni channel retailer, that is, a retailer that sells items both via brick-and-mortar stores and online. (Zadeh, L.(1965))

In the crisp environment, all the parameters related to the model such as consumable cost, employee cost, security cost, wastage cost, admin cost, marketing cost, demand rate are known and have uncertain value. While some trading scenarios apply to such conditions, in fact most scenarios and parameters and variables are very uncertain in fast changing market conditions. These parameters and variables are referred to as ambiguous parameters in such cases. Clarification acknowledges the reliability of the model by allowing ambiguity throughout the system, which brings it closer to reality.

A variety of methods have been proposed in the literature for sorting obscure numbers. Mandal, N.K.(2012) proposed vague numerical cost parameters. Asma, F., Henry, E.C., Amirthara. (2015) decribed the fuzzy inventory model subject to constraints has been transformed in to the crisp inventory problem using Robust's ranking indices. Kasthuri, R., Vasanthi, P., Ranganayaki, S., Seshaiah, C. V. (2011) developed with three constraints and have been solved by Kuhn tucker conditions. Roy, T.K., Maiti, M.(1998) explored existing problems in their solution procedure for Kuhn–Tucker's method . Although there are some comparative studies, it is not yet known whether similar ranking methods are still in use today, and they have the potential to introduce an ambiguous set of ranking asteroids. The obscure inventory sample for factory sales outlets was obtained using the ranking asteroid ambiguous set. (Dhanam, K., Parimaladevi, M.(2016))

Till now, there is no literature by using asteroid fuzzy sets. This paper developed the fuzzy inventory model by using asteroid fuzzy set. In a realistic situation, the total expenditure for an inventory model and the investment amount may be limited. The inventory costs, consumable cost, employee cost, security cost, wastage cost, admin cost, marketing cost may be flexible with some vagueness for their values. The ambiguity of the above parameters necessitates analyzing the inventory problem in a fuzzy environment. The inventory of multiple items for factory outlets is the subject of this article. The asteroid ambiguous is used to represent the cost parameters. The model is distorted by the ranking system, which determines the average total cost. Finally, a numerical example of the sample and sensitivity analysis is given

PRELIMINARIES

Definition (Consumable cost)

The cost associated with consumer goods (e.g., food and clothing) used for everyday life is used up or exhausted during their consumption, retailers often make more profit by selling consumer goods ranging from non-recycled staples. The demand for this specialty has increased as a consequence of current corporate scandals.

Definition (Security cost)

It means the cost of reimbursing the City for ordinary, necessary and reasonable direct costs of providing security services within the District.

Definition (Employee cost)

The total costs of hiring a person are known as employee expenses. It varies depending on the country, industry, and profession.

Definition (Admin cost)

Admin costs are the costs incurred only by running a business or hotel called overhead costs or fixed costs. Examples of administrative expenses include taxes, rent, insurance, license fees, utilities, accounting and legal boards, administrative staff, and facility maintenance.

Definition (Marketing cost)

Cost associated with marketing is the system of selling an item. For example, determining its price, the areas in which it should be offered and how it should be advertised. The cost of marketing explains the cost of changing the title and the cost of moving the goods to the customer.

Definition (Wastage cost)

Waste is the amount of raw material lost in the production process. This may include losses due to shrinkage, scraping or evaporation. When the actual amount of waste exceeds the standard waste cost, it appears as a negative amount or utility variance in the business expense statement.

Definition (Factory outlet)

Factory Outlet, also known as factory Shop, is a store where manufacturers sell their products directly to the public at steep discounts. The Factory outlet center is a manufacturer-owned store that sells shares of the company directly to the public. A factory stores inventory could be first class inventory or cancelled, irregular, cancelled orders with extremely low prices.

ASSUMPTION AND NOTATIONS

Assumption

i. Multi item will be considered.

ii. Demand rate is uniform.

iii. Shortages are not allowed

iv. Time horizon is finite.

v. The production rate is always greater than demand rate.

vi. Investment constraint allowed.

Notation

The following are for the i^{th} item (i= 1,2,3,….N)

N	-	No. of items
Q_i	-	Outlet quantity (Decision variable)
q_i	-	Sales quantity (Decision variable)
R_i	-	Demand is constant
C_{c_i}	-	Consumable cost per unit per unit time
S_{c_i}	-	Security cost per unit per unit time
E_{c_i}	-	Employee cost
A_{c_i}	-	Admin cost
M_{c_i}	-	Marketing cost
W_{c_i}	-	Wastage cost
B_i	-	Price per unit of item
I_m	-	Maximum investment
T_c(or) C (q_i,Q_i)	-	Total cost per unit time
$\tilde{C}_{c_i}$	-	Fuzzy Consumable cost per unit per unit time
$\tilde{S}_{c_i}$	-	Fuzzy Security cost per unit per unit time
$\tilde{E}_{c_i}$	-	Fuzzy Employee cost
$\tilde{A}_{c_i}$	-	Fuzzy Admin cost
$\widetilde{M}_{c_i}$	-	Fuzzy Marketing cost
$\widetilde{W}_{c_i}$	-	Fuzzy Wastage cost
$\tilde{B}_i$	-	Fuzzy Price per unit of item
$\ddot{I}_m$	-	Fuzzy maximum investment
$\tilde{T}_c$ (or) $\tilde{C}(q_i,Q_i)$	-	Fuzzy Total cost per unit time

The factory outlet model is constructed by using the above assumptions and notations.

MATHEMATICAL MODEL IN CRISP ENVIRONMENT

$$\text{Total cost} = \sum_{i=1}^{N} [\text{Consumable cost} + \text{Security cost} + \text{Employee cost} + \text{Admin cost} + \text{Marketing cost} + \text{Wastage cost}]$$

$$T_C(or)C(q_i,Q_i) = \sum_{i=1}^{n}\left(\frac{1}{2}C_{C_i}\frac{q_i^{\,2}}{Q_i} + \frac{1}{2}S_{C_i}\frac{q_i^{\,2}}{Q_i} + \frac{E_{C_i}R_i}{Q_i} + \frac{A_{C_i}R_i}{Q_i} + \frac{M_{C_i}R_i}{Q_i} + \frac{1}{2}W_{C_i}\frac{(Q_i-q_i)^2}{Q_i}\right) \tag{1}$$

The problem is stated that minimize the total cost (TC), subject to constraint

$$\sum_{i=1}^{n} B_i Q_i \le I_m$$

Minimize TC

$$\sum_{i=1}^{n} B_i Q_i \le I_m \tag{2}$$

Using Lagrange multipliers method, the Lagrange function is

$$L(q_i,Q_i,\lambda) = \sum_{i=1}^{N}\left(\frac{1}{2}C_{C_i}\frac{q_i^{\,2}}{Q_i} + \frac{1}{2}S_{C_i}\frac{q_i^{\,2}}{Q_i} + \frac{E_{C_i}R_i}{Q_i} + \frac{A_{C_i}R_i}{Q_i} + \frac{M_{C_i}R_i}{Q_i} + \frac{1}{2}W_{C_i}\frac{(Q_i-q_i)^2}{Q_i}\right) - \lambda\left(\sum_{i-1}^{N} B_i Q_i - I_m\right) \tag{3}$$

By using Kuhn-Tucker necessary condition in (3)

Differentiate the equation (3) with respect to q_i and equal to zero

ie) $\dfrac{\partial L(q_i, Q_i, \lambda)}{\partial q_i} = 0$

$$q_i = \left(\frac{W_{C_i}}{C_{C_i} + S_{C_i} + W_{C_i}} Q_i \right) \quad , \qquad i=1,2,3\ldots N \tag{4}$$

Differentiate the equation (3) with respect to Q_i and equal to zero

ie) $\dfrac{\partial L(q_i, Q_i, \lambda)}{\partial Q_i} = 0$

$$W_{C_i} Q_i^{2} - 2\lambda s_i Q_i^{2} = (C_{C_i} + S_{C_i} + W_{C_i}) q_i^{2} + 2R_i (E_{C_i} + A_{C_i} + M_{C_i}) \tag{5}$$

Substitute the expression $q_i = \left(\dfrac{W_{C_i}}{C_{C_i} + S_{C_i} + W_{C_i}} Q_i \right)$ in the equation (5)

ie) $$Q_i^* = \sqrt{C_{C_i} + S_{C_i} + W_{C_i}} \sqrt{\frac{2R_i (E_{C_i} + A_{C_i} + M_{C_i})}{W_{C_i}(C_{C_i} + S_{C_i}) - 2\lambda B_i (C_{C_i} + S_{C_i} + W_{C_i})}} \tag{6}$$

Substitute equation (6) in (4)

$$q_i = \left[\frac{W_{C_i}}{C_{C_i} + S_{C_i} + W_{C_i}} \left[\sqrt{C_{C_i} + S_{C_i} + W_{C_i}} \sqrt{\frac{2R_i (E_{C_i} + A_{C_i} + M_{C_i})}{W_{C_i}(C_{C_i} + S_{C_i}) - 2\lambda B_i (C_{C_i} + S_{C_i} + W_{C_i})}} \right] \right]$$

$$q_i^* = \left[\sqrt{\frac{W_{C_i}}{C_{C_i} + S_{C_i} + W_{C_i}}} \sqrt{\frac{2R_i (E_{C_i} + A_{C_i} + M_{C_i}) W_{C_i}}{W_{C_i}(C_{C_i} + S_{C_i}) - 2\lambda B_i (C_{C_i} + S_{C_i} + W_{C_i})}} \right]$$

Differentiate the equation (3) with respect to λ and equal to zero

ie) $\dfrac{\partial L(q_i, Q_i, \lambda)}{\partial \lambda} = 0$

$$\frac{\partial L(q_i, Q_i, \lambda)}{\partial \lambda} = \sum_{i=1}^{n} \left(B_i Q_i - I_m \right) = 0$$

$$\left(B_i \sqrt{C_{C_i} + S_{C_i} + W_{C_i}} \sqrt{\frac{2R_i (E_{C_i} + A_{C_i} + M_{C_i})}{W_{C_i}(C_{C_i} + S_{C_i}) - 2\lambda B_i (C_{C_i} + S_{C_i} + W_{C_i})}} \right) = 0$$

(7)

Substitute the expression of $Q_i *$ and $q_i *$ in equation (1), the minimum average cost is derived

$$C(q_i, Q_i) = \sum_{i=1}^{n} \left[\left(\frac{W_{C_i}}{C_{C_i} + S_{C_i} + W_{C_i}} \right)^2 \frac{Q_i^{2}}{2Q_i} (C_{C_i} + S_{C_i}) + \frac{R_i}{Q_i} (E_{C_i} + A_{C_i} + M_{C_i}) + \frac{1}{2} W_{C_i} \left[Q_i - \left(\frac{W_{C_i}}{C_{C_i} + S_{C_i} + W_{C_i}} \right) Q_i \right]^2 \right]$$

$$\Rightarrow C(q_i, Q_i) = \sum_{i=1}^{n} \left[\sqrt{\frac{(C_{C_i} + S_{C_i}) W_{C_i}}{C_{C_i} + S_{C_i} + W_{C_i}}} + \sqrt{2R_i (E_{C_i} + A_{C_i} + M_{C_i})} \right] \tag{8}$$

ASTEROID FUZZY SET

Definition and its properties

An Asteroid fuzzy set $\tilde{A}$ described as a fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} w\left[1+\left(\dfrac{x-a}{b-a}\right)^{\frac{3}{2}}\right] & a \le x \le b \\ w\left[1+\left(\dfrac{x-c}{b-c}\right)^{\frac{3}{2}}\right] & b \le x \le c \\ \alpha - base \quad x = w \\ w\left[1-\left(\dfrac{x-c}{b-c}\right)^{\frac{3}{2}}\right] & c \le x \le b \\ w\left[1-\left(\dfrac{x-a}{b-a}\right)^{\frac{3}{2}}\right] & b \le x \le a \end{cases}$$

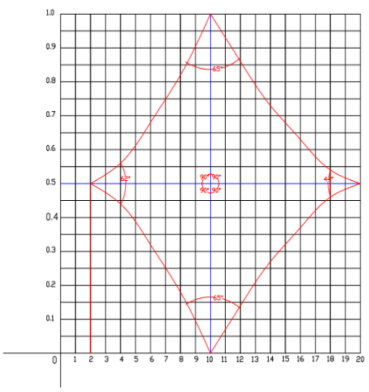

Figure 1: Graphical representation of asteroid fuzzy set where w = 0.5

Where 0.1≤ w ≤ 0.5 and a, b, c, are real numbers.

This type of fuzzy set be denoted as $\tilde{A}$=[a,b,c; ω]

Properties of asteroid fuzzy set

$\mu_{\tilde{A}}(x)$ satisfies the following conditions:

1. $\mu_{\tilde{A}}(x)$ is a continuous function from R to the closed interval [0,1].
2. $\mu_{\tilde{A}}=0$, $0<x\le a$
3. $\mu_{\tilde{A}}=L(x)$ is strictly increasing and decreasing on (a, b)
4. $\mu_{\tilde{A}}=R(x)$ is strictly decreasing and increasing on (b ,c)
5. $\mu_{\tilde{A}}=1, x=b$.

Ranking asteroid fuzzy set of cost parameters with best approximation level

Let $\tilde{A}$ = (a,b,c) be a Asteroid fuzzy set. The α - level interval of $\tilde{A}$ is defined as $A_\alpha \in [A_L(\alpha), A_R(\alpha)]$. When $\tilde{A}$ is a fuzzy set , the left and right α cuts are

$$A_L(\alpha) = \begin{cases} a+(b-a)\left(1-\frac{\alpha}{w}\right)^{\frac{2}{3}} & if\, 0 \le \alpha \le w \\ a+(b-a)\left(\frac{\alpha}{w}-1\right)^{\frac{2}{3}} & if w \le \alpha \le 2w \end{cases}$$

$$C_L(\alpha) = \frac{\int_0^{2w} A_L(\alpha) f(\alpha) d\alpha}{\int_0^{2w} f(\alpha) d\alpha}$$

$$C_L(\alpha) = \frac{\int_0^{w} C_{L_1}(\alpha) f(\alpha) d\alpha + \int_w^{2w} C_{L_2}(\alpha) f(\alpha) d\alpha}{\int_0^{w} f(\alpha) d\alpha + \int_w^{2w} f(\alpha) d\alpha}$$

$$C_L(\alpha) = \frac{\int_0^{w} a+(b-a)\left(1-\frac{\alpha}{w}\right)^{\frac{2}{3}} .d\alpha + \int_w^{2w} a+(b-a)\left(\frac{\alpha}{w}-1\right)^{\frac{2}{3}} \alpha d\alpha}{\int_0^{w} 1 d\alpha + \int_w^{2w} \alpha d\alpha}$$

$$C_L(\alpha) = \frac{2}{2+3w}\left\{a\left(1+\frac{3}{2}w\right)+\frac{3}{5}(b-a)\left(1+\frac{13}{8}w\right)\right\} \tag{9}$$

$$A_R(\alpha) = \begin{cases} c+(b-c)\left(1-\frac{\alpha}{w}\right)^{\frac{2}{3}} & if\, 0 \le \alpha \le w \\ c+(b-c)\left(\frac{\alpha}{w}-1\right)^{\frac{2}{3}} & if w \le \alpha \le 2w \end{cases}$$

$$C_R(\alpha) = \frac{\int_0^{2w} A_R(\alpha) f(\alpha) d\alpha}{\int_0^{2w} f(\alpha) d\alpha}$$

$$C_R(\alpha) = \frac{\int_0^{w} C_{R_1}(\alpha) f(\alpha) d\alpha + \int_w^{2w} C_{R_2}(\alpha) f(\alpha) d\alpha}{\int_0^{w} f(\alpha) d\alpha + \int_w^{2w} f(\alpha) d\alpha}$$

$$C_R(\alpha) = \frac{\int_0^{w} c+(b-c)\left(1-\frac{\alpha}{w}\right)^{\frac{2}{3}} .d\alpha + \int_w^{2w} c+(b-c)\left(\frac{\alpha}{w}-1\right)^{\frac{2}{3}} \alpha d\alpha}{\int_0^{w} 1 d\alpha + \int_w^{2w} \alpha d\alpha}$$

$$C_R(\alpha) = \frac{2}{2+3w}\left\{c\left(1+\frac{3}{2}w\right)+\frac{3}{5}(b-c)\left(1+\frac{13}{8}w\right)\right\} \tag{10}$$

Ranking Asteroid fuzzy set of cost parameters with best approximation level is

$R_\alpha(\tilde{C}) = \alpha C_L(\alpha) + (1-\alpha) C_R(\alpha)$

Using the equations (9) & (10)

$$R_\alpha(\tilde{C}) = \frac{2}{2+3w}\left\{\left(1+\frac{3}{2}w\right)\left[\alpha(a-c)+c\right] - \frac{3}{5}\left(1+\frac{13}{8}w\right)\left[\alpha(a-c)-(b-c)\right]\right\}$$

$$\frac{2}{2+3w}\left\{\left(1+\frac{3}{2}w\right)(\alpha a - c\alpha + c) - \frac{3}{5}\left(1+\frac{13}{8}w\right)\left[\alpha a - \alpha c - b + c\right]\right\}$$

INVENTORY MODEL IN FUZZY ENVIRONMENT

The above crisp model (4.1) is fuzzified by asteroid fuzzy set, then

$$\tilde{C}(\tilde{q}_i, \tilde{Q}_i) = \sum_{i=1}^{n}\left(\frac{1}{2}\tilde{C}_{C_i}\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i} + \frac{1}{2}\tilde{S}_{C_i}\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i} + \frac{\tilde{E}_{C_i}\tilde{R}_i}{\tilde{Q}_i} + \frac{\tilde{A}_{C_i}\tilde{R}_i}{\tilde{Q}_i} + \frac{\tilde{M}_{C_i}\tilde{R}_i}{\tilde{Q}_i}{}_i + \frac{1}{2}\tilde{W}_{C_i}\frac{(\tilde{Q}_i - \tilde{q}_i)^2}{\tilde{Q}_i}\right)$$

Subject to

$$\sum_{i=1}^{n}\tilde{B}_i\tilde{Q}_i \le \tilde{I}_m$$

Minimize TC

$$\sum_{i=1}^{n}\tilde{B}_i\tilde{Q}_i \le \tilde{I}_m$$

The total cost is defuzzified by ranking method

$$R_{\tilde{C}}(\alpha, \tilde{q}_i, \tilde{Q}_i, \lambda) = \sum_{i=1}^{n}\left(\begin{array}{l}\frac{1}{2}R_{\tilde{C}_{C_i}}(\alpha)\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i} + \frac{1}{2}R_{\tilde{S}_{C_i}}(\alpha)\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i} + \frac{R_{\tilde{E}_{C_i}}(\alpha)\tilde{R}_i}{\tilde{Q}_i} + \frac{R_{\tilde{A}_{C_i}}(\alpha)\tilde{R}_i}{\tilde{Q}_i} + \frac{R_{\tilde{M}_{C_i}}(\alpha)\tilde{R}_i}{\tilde{Q}_i} + \\ \frac{1}{2}R_{\tilde{W}_{C_i}}(\alpha)\frac{(\tilde{Q}_i - \tilde{q}_i)^2}{\tilde{Q}_i}\end{array}\right) - \lambda\left(\sum_{i-1}^{N}\tilde{B}_i\tilde{Q}_i - \tilde{I}_m\right)$$

$$R_{\tilde{C}}(\alpha, \tilde{q}_i, \tilde{Q}_i, \lambda) = \sum_{i=1}^{n}\left(\begin{array}{l}\frac{1}{2}\left(\alpha\tilde{C}_{C_{i_L}} + (1-\alpha)\tilde{C}_{C_{i_R}}\right)\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i} + \frac{1}{2}\left(\alpha\tilde{S}_{C_{i_L}} + (1-\alpha)\tilde{S}_{C_{i_R}}\right)\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i} \\ + \frac{\left(\alpha\tilde{E}_{C_{i_L}} + (1-\alpha)\tilde{E}_{C_{i_R}}\right)\tilde{R}_i}{\tilde{Q}_i} + \frac{\left(\alpha\tilde{A}_{C_{i_L}} + (1-\alpha)\tilde{A}_{C_{i_R}}\right)\tilde{R}_i}{\tilde{Q}_i} \\ + \frac{\left(\alpha\tilde{M}_{C_{i_L}} + (1-\alpha)\tilde{M}_{C_{i_R}}\right)\tilde{R}_i}{\tilde{Q}_i} + \frac{1}{2}\left(\alpha\tilde{W}_{C_{i_L}} + (1-\alpha)\tilde{W}_{C_{i_R}}\right)\frac{(\tilde{Q}_i - \tilde{q}_i)^2}{\tilde{Q}_i}\end{array}\right) - \lambda\left(\sum_{i-1}^{N}\tilde{B}_i\tilde{Q}_i - \tilde{I}_{m_i}\right) \quad (11)$$

Differentiate the equation (11) with respect to α

$$\frac{\partial R_{\tilde{C}}(\alpha, \tilde{q}_i, \tilde{Q}_i, \lambda)}{\partial\alpha} = \frac{1}{2}\left(\tilde{C}_{C_{i_L}} - \tilde{C}_{C_{i_R}}\right)\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i} + \frac{1}{2}\left(\tilde{S}_{C_{i_L}} - \tilde{S}_{C_{i_R}}\right)\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i} + \left(\tilde{E}_{C_{i_L}} - \tilde{E}_{C_{i_R}}\right)\frac{\tilde{R}_i}{\tilde{Q}_i} + \left(\tilde{A}_{C_{i_L}} - \tilde{A}_{C_{i_R}}\right)\frac{\tilde{R}_i}{\tilde{Q}_i}$$

$$+\left(\tilde{M}_{C_{i_L}} - \tilde{M}_{C_{i_R}}\right)\frac{\tilde{R}_i}{\tilde{Q}_i} + \frac{1}{2}\left(\tilde{W}_{C_{i_L}} - \tilde{W}_{C_{i_R}}\right)\left(\frac{Q_i - \tilde{R}_i}{\tilde{Q}_i}\right)^2 = 0$$

Differentiate the equation (11) with respect to $\tilde{q}_i$

$$\frac{\partial R_{\tilde{C}}(\alpha, \tilde{q}_i, \tilde{Q}_i, \lambda)}{\partial\tilde{q}_i} = \left(\alpha\tilde{C}_{C_{i_L}} + (1-\alpha)\tilde{C}_{C_{i_R}}\right)\frac{\tilde{q}_i}{\tilde{Q}_i} + \left(\alpha\tilde{S}_{C_{i_L}} + (1-\alpha)\tilde{S}_{C_{C_{i_R}}}\right)\frac{\tilde{q}_i}{\tilde{Q}_i}$$

$$-\left(\alpha\tilde{W}_{C_{i_L}} + (1-\alpha)\tilde{W}_{C_{i_R}}\right)\frac{(\tilde{Q}_i - \tilde{q}_i)}{\tilde{Q}_i} = 0$$

Differentiate the equation (11) with respect to $\tilde{Q}_i$

$$\frac{\partial R_{\tilde{C}}(\alpha,\tilde{q}_i,\tilde{Q}_i,\lambda)}{\partial \tilde{Q}_i} = -\frac{1}{2}\left[\alpha\tilde{C}_{C_{i_L}} + (1-\alpha)\tilde{C}_{C_{i_R}}\right]\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i^{\,2}} - \frac{1}{2}\left[\alpha\tilde{S}_{C_{i_L}} + (1-\alpha)\tilde{S}_{C_{i_R}}\right]\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i^{\,2}}$$

$$-\left[\alpha\tilde{E}_{C_{i_L}} + (1-\alpha)\tilde{E}_{C_{i_R}}\right]\frac{\tilde{R}_i}{\tilde{Q}_i^{\,2}} - \left[\alpha\tilde{A}_{C_{i_L}} + (1-\alpha)\tilde{A}_{C_{i_R}}\right]\frac{\tilde{R}_i}{\tilde{Q}_i^{\,2}}$$

$$-\left[\alpha\tilde{M}_{C_{i_L}} + (1-\alpha)\tilde{M}_{C_{i_R}}\right]\frac{\tilde{R}_i}{\tilde{Q}_i^{\,2}} + \frac{1}{2}\left[\alpha\tilde{W}_{C_{i_L}} + (1-\alpha)\tilde{W}_{C_{i_R}}\right]\left[\frac{2\tilde{Q}_i(\tilde{Q}_i-\tilde{q}_i)-(\tilde{Q}_i-\tilde{q}_i)^2}{\tilde{Q}_i^{\,2}}\right] - \lambda(\tilde{B}_i - \tilde{I}_m) = 0$$

Differentiate the equation (11) with respect to λ

$$\frac{\partial R_{\tilde{C}}(\alpha,\tilde{q}_i,\tilde{Q}_i,\lambda)}{\partial \lambda} = -\frac{1}{2}\left[\alpha\tilde{C}_{C_{i_L}} + (1-\alpha)\tilde{C}_{C_{i_R}}\right]\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i^{\,2}} - \frac{1}{2}\left[\alpha\tilde{S}_{C_{i_L}} + (1-\alpha)\tilde{S}_{C_{i_R}}\right]\frac{\tilde{q}_i^{\,2}}{\tilde{Q}_i^{\,2}}$$

$$-\left[\alpha\tilde{E}_{C_{i_L}} + (1-\alpha)\tilde{E}_{C_{i_R}}\right]\frac{\tilde{R}_i}{\tilde{Q}_i^{\,2}} - \left[\alpha\tilde{A}_{C_{i_L}} + (1-\alpha)\tilde{A}_{C_{i_R}}\right]\frac{\tilde{R}_i}{\tilde{Q}_i^{\,2}} - \left[\alpha\tilde{M}_{C_{i_L}} + (1-\alpha)\tilde{M}_{C_{i_R}}\right]\frac{\tilde{R}_i}{\tilde{Q}_i^{\,2}}$$

$$+\frac{1}{2}\left[\alpha\tilde{W}_{C_{i_L}} + (1-\alpha)\tilde{W}_{C_{i_R}}\right]\left[\frac{2\tilde{Q}_i(\tilde{Q}_i-\tilde{q}_i)-(\tilde{Q}_i-\tilde{q}_i)^2}{\tilde{Q}_i^{\,2}}\right] - (\tilde{B}_i\tilde{Q}_i - \tilde{I}_{m)} = 0$$

NUMERICAL EXAMPLE

Develop a mathematical program to minimize the average total cost. Consider a factory outlet shop which produces three type of items. The three items are readymade, carpets and suits. The appropriate data given,

Items	1	2	3		
$\tilde{S}_{C_i}$	(25, 30, 35)	(40, 42, 44)	(51, 54, 57)		
$\tilde{C}_{C_i}$	(10, 11, 12)	(13, 14, 15)	(16, 17, 18)		
$\tilde{E}_{C_i}$	(100, 130, 160)	(150, 151, 159)	(170, 180, 190)		
$\tilde{A}_{C_i}$	(140, 145, 150)	(140,150,160)	(200,220,240)		
$\tilde{M}_{C_i}$	(110, 115, 120)	(112, 113, 114)	(130, 136, 142)		
$\tilde{W}_{C_i}$	(5, 5.5 ,6)	(7, 7.1, 8)	(9, 9.2, 9.4)		
$\tilde{B}_i$	2	3	4	Rs.	per unit item

$\tilde{I}_m$ = Rs. 2250 w = 0.5 and, S_w = 190, i = 1,2,3.

Using MATLAB software, the optimal values Q*,q*,α*, λ* and T_c* are tabulated.

Table 1: Optimal Solution

Model	Item	C_c	S_c	A_c	E_c	M_c	W_c	B	R	Q*	q*	λ*	T_c*
Crisp 1	**1**	25	10	140	100	110	8	2	200	190.08	23.76		3440.3
	2	40	13	140	150	112	10	3	250	191.46	22.34		
	3	51	16	200	170	130	12	4	300	212.46	29.42		
Crisp 2	**1**	30	11	145	130	115	105	2	200	194.12	22.96		3630.3
	2	42	14	150	151	120	121	3	250	195.77	22.03		
	3	54	17	220	180	136	14	4	300	214.38	24.59		
Crisp 3	**1**	35	12	150	160	120	12	2	200	197.38	22.34		3828.5
	2	44	15	160	159	128	14	3	250	199.21	21.67		
	3	57	18	240	190	142	16	4	300	221.16	23.93		

Table 2: Sensitivity Analysis (Ranking method)

Fuzzy	C_{C_L}	C_{C_R}	S_{C_L}	S_{C_R}	A_{C_L}	A_{C_R}	E_{C_L}	E_{C_R}	M_{C_L}	M_{C_R}	W_{C_L}	W_{C_R}	B	**R**	**Q***	**q***	**α***	**λ***	**T_c***
Item 1	28.1071	31.8929	10.6214	11.3786	143.1071	146.8929	118.6429	141.3571	113.1071	116.8929	5.3107	5.6893	2	200	179.5992	67.7601			
Item 2	41.2429	42.7571	13.6214	14.3786	146.2143	153.7857	150.6214	154.0286	116.9714	123.0286	7.0621	7.4407	3	250	180.0351	62.3570	0.8512	0.9863	3390.04
Item 3	52.8643	55.1357	16.6214	17.3786	212.4286	227.5714	176.2143	183.7857	133.7286	138.2714	9.1243	9.2757	4	300	199.1973	60.6927			

OBSERVATION

In table 1 shows the optimal values for ambiguous models and smooth models. Since our allowable spending range is (Rs. 3000 to Rs. 3700) only the two crisp model costs in Table 1 fall within this range, although these values are higher than the ambiguous models. The average total cost obtained in the ambiguous model is lower than that obtained in the crisp model, as shown off in the table above. In comparison to the crisp model, the fuzzy model is more effective.

CONCLUSION

In this paper, it developed a fuzzy inventory model for multi-item in our numerical experiments, The inventory level in the fuzzy environment is high compared to the crisp value, for the fuzzy inventory model with investment constraint, Moreover, the fuzzy inventory model subject to the constraints has been transformed in to crisp inventory problem using ranking indices. Numerical example shows that by this method we can have the optimal total cost. Ranking asteroid fuzzy set method we have shown that the total cost obtained is optimal. Moreover, one can conclude that the solution of fuzzy problems can be obtained by ranking method effectively. The minimum total cost in the crisp environment is high compared to the fuzzy value. Finally, conclude that the fuzzy model can be executable in the real work.

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