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Fuzzy Linear programming approach in Water Pollution Control

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Abstract

Purpose of the study: The purpose of the study is to calculate the water pollution level of a source. Because water pollution is a very big problem in front of us. Mining and the related activities are responsible for large scale water pollution.

Methodology: The problem of water pollution control (WPC) may be treated as a multiple objective decision making problem. This paper deals with use of fuzzy linear programming in water pollution control problem. Min operator has been used to construct the overall achievement function. A semi hypothetical case has been studied with regard to mining vis-a-vis water pollution.

Main Findings: After calculation and using all the methods the main finding is that total WPC cost is 54411.12 units.

Applications of this study: We can use this method of water purification to purify the big source of water supply. After using this method the cost of water purification can be calculated and minimized.

Novelty/Originality of this study: This calculation is based on the secondary data source and this can help the system to use the proper method and calculation of expense in purifying water.

INTRODUCTION

The extent of water pollution caused by mining and allied industries depends upon the composition of mineral being mined including its processing, overburden characteristics, nature of the water body etc. These activities may cause large scale pollution. The level of total suspended solid (TSS) may be high, other parameters such as, coliform number may increase which indicate the bio-chemical degradation of water. The pollution due to organic discharge is also there. The treatment of water at present is limited to effluents from coke plants, washeries and water for domestic use, etc. In order to preserve aquatic life, it is important to have minimum levels of BOD (Burnwal, A.P. & Dey, U.K. (2002), Revelle, C.S., Loucks, D.P. and Lynn, W.R.(1968), Zimmermann, H. J. (1978)).

In practice to solve the optimization problems, the techniques such as linear programming, nonlinear programming, goal programming, integer programming, geometric programming, fractional programming, dynamic programming are required. If either some/ all variables , any/ all constraints or any/ all objective functions in the optimization problem are imprecise then fuzzy mathematical programming approach plays a key role i.e. the need of fuzzy modeling arises when we try to deal with imprecise decision situations(Sobel, M.J.(1965), Yu, P., and Chen, C. (2000)). In the problem of WPC some parameters are not precisely defined so in this paper the technique of linear programming in fuzzy environment has been used to solve a typical water pollution control problem using min operator(Fitch, W.N., King P.H. and Young, G.K.(1970)).

LITERATURE REVIEW

Review of literature can be written as per the requirement of your study i.e. argumentive or systematic or methodological related to the work of previous researchers should be presented. For help see the link - https://libguides.usc.edu/writingguide/literaturereview

METHODOLOGY

Let us consider a hypothetical situation of WPC where a small tributary to a main river having 'k' sources at the same number of different locations L_1, L_2 …….., L_k which are discharging b_1, b_2, ……..b_k kg of biological oxygen demand (BOD) /Unit. The flow in the river is v m^3/sec. The velocity of flow in the river is such that the travel time from L_1 location to L_2 location is t_1 units, from L_2 location to L_3 location is t_2 units, from L_3 location to L_4 location t_3, units, ………………. from L_{k1} location L_k location is t_{k-1}, units. From L_k location to mouth of tributary is t_k unit. The rate of BOD exertion is 'r' mg/unit. To remove one kg of BOD at the above locations, costs are C_1, C_2, ………C_k units respectively. The degree of treatment at any location is about D%. The requirement as per the pure water quality standards is that the BOD in the river should not exceed 'd' mg/liter at any location. Since the rate of BOD execrtation is 'α'mg/units. So due to self purification f , fraction of water pollution remaining after 't' unit is described as $f = e^{-\alpha t}$ The values of 'f' are V_1, V_2, V_3, ………, V_K, corresponding to t_1, t_2, …….,t_k respectively. At location L_1, b_1 unit of BOD are discharged into the flow. After a travel time of t_1 unit, BOD in the river at L_2 location due to discharge at location L_1

will have been reduced through self purification to b_1'. But at L_2 location, new discharge of b_2 kg will lead to concentration of b_2'. Similarly concentration at locations L_3, …….,L_k may determined. Let x_1 is the unknown fraction of BOD removed at L_1 location. Then $(1-x_1)$ is the fraction BOD discharged in the stream, Similarly x_2, ………,x_k are defined. Let us assume that water quality standards require that the BOD in the river should not exceed 'q'mg/ltr. at any place.

A Linear programming model of the problem is given as:

Min

$$b_1 c_1 x_1 + b_2\, c_2 x_2 + \ldots \ldots .. + b_k\, c_k\, x_k$$

subject to

$$a_{11}\,(1-x_1) \quad \le q$$

$$a_{21}\,(1-\,x_1) + \,a_{22}\,(1-\,x_2) \le q$$

$$a_{31}\,(1-\,x_1) + \,a_{32}\,(1-\,x_2) + \,(a_{33}\,(1-\,x_3) \le \mathrm{q}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{k1}(1-\,x_1) + \,a_{k2}\,(1-\,x_2) + \,\ldots. \,\,+ a_{kk}(1-\,x_k) \le q$$

$$x_1 \;\le D, x_2 \le D, x_3 \le D, \ldots \ldots \ldots \ldots \ldots \ldots x_k \le D$$

$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, \ldots \ldots \ldots \ldots \ldots\; x_k \ge 0, \ldots\ldots\ldots(2.1)$$

Where $a_{11}, a_{21}, a_{22}, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots\; a_{kk}$ are evaluated with help of discharging of BOD and its self purification at different locations. Model(2.1) is solved by linear programming algorithm.

SAMPLE PROBLEM

Assuming that small tributary to a main river having four different locations L_1, L_2, L_3 & L_4 which are discharging 4000, 8000, 3500 & 3000 kg of biological oxygen demand (BOD)/day. The flow in the river is 1.16 m^3/sec. The velocity of flow in the river such that the travel time from L_1 location to L_2 location is 1 unit, from L_2 location L_3 location is about one days, from L_3 location to L_4 location is 1 unit and from L_4 location to mouth of the tributary is 1 unit. The rate of BOD exertion is 0.16 mg/unit. To remove one kg of BOD at the above locations costs are 6, 4, 5 and 2 units respectively. The degree of treatment at any location is about 90%. The requirement as per the water quality standards is that the BOD in the river should not exceed 30 mg per liter at any location.

The rate of BOD exceration is α=0.16 mg per unit. So due to self purification, f, fraction of water pollution remaining after 't' units is described as f=$e^{-\alpha t}$. The values of f are 0.85214, 0.78662, 0.72614 & 0.82530 corresponding to t=1, 1.5, 2 & 1.2 units respectively. At L_1 location, 4000 kg of BOD are discharged into a flow of 1m^3/sec which leads to a BOD concentration of 40 mg/liter.

After a travel time of 1 unit, the BOD in the river at L_2 location due to discharge at L_1 location will have been reduced through self purification to 40 x .85214 =34.0856 mg/liter. But at L_2 location, new discharge of 8000 kg will lead to concentration of 80 mg/liter. Similarly, concentrations at other L_3 locations may be determined. Let x_1 be the unknown fraction of BOD removed at L_1 location. Then $(1-x_1)$ is the fraction of BOD discharged in the stream. Similarly x_2, x_3 & x_4 are defined. Here we may write an inequation for L_1 location as;

$40\,(1-x_1) \le 30$ or $x_1 \ge 0.25$

Similarly for other three locations the inequations can be written as:

For location L_2, it is

$$34.0856\, x_1 + 80\, x_2 \;\ge 84.0856$$

For location L_3, it is

$$29.0457\, x_1 + \,68.172\, x_2 + \,35\, x_3 \;\ge 102.2177$$

For location L_4 it is

$$24.751 x_1 + \,58.09209\, x_2 + \,29.8249\, x_3 + 30\, x_4 \;\ge 112.66799$$

The WPC cost will be C= $24000x_1 + 32000x_2 + 17500x_3 + 6000x_4$. The maximum possible degree of treatment at each location is 90%.

The LP model is given as:

Determine x_1, x_2, x_3 & x_4 that

Minimizes $24000x_1 + 32000x_2 + 17500x_3 + 6000x_4$

subject to

$x_1 \geq 0.25$

$34.0856x_1 + 80x_2 \geq 84.0856$

$29.0457x_1 + 68.172x_2 + 35x_3 + \geq 102.2177$

$24.751x_1 + 58.09209x_2 + 29.8249x_3 + 30x_4 \geq 112.66799$

$x_1 \leq .9$

$x_2 \leq .9$

$x_3 \leq .9$

$x_4 \leq .9$

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \; x_4 \geq 0$ …………………… (2.2)

The model (2.2) is solved by simplex method. The solution is as:

$x_1 = .354566$, $x_2 = .9$, $x_3 = .873265$, $x_4 = .852140$, Min cost of WPC= 57704.570 units.

FUZZY MODEL OF THE PROBLEM:

Determine x_1, x_2, x_3 & x_4 that

Minimizes

Subject to $24000x_1 + 32000x_2 + 17500x_3 + 6000x_4 \underset{\sim}{\geq} 6000$

$x_1 \geq 0.25$

$34.0856x_1 + 80x_2 \geq 84.0856$

$29.0457x_1 + 68.172x_2 + 35x_3 \geq 102.2177$

$24.751x_1 + 58.09209x_2 + 29.8249x_3 + 30x_4 \geq 112.66799$

$x_1 \underset{\sim}{\leq} .9$

$x_2 \underset{\sim}{\leq} .9$

$x_3 \underset{\sim}{\leq} .9$

$x_4 \underset{\sim}{\leq} .9$

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \; x_4 \geq 0$ …………………… (2.3)

Where, the symbol $\sim$ stands for fuzziness

The model (2.3) is solved by simplex technique. To solve, it is required to defuzzify each fuzzy goal using linear membership functions $\mu_1, \mu_2, \mu_3, \mu_4$ & μ_5 with help of tolerant limits

$$\mu_1 = (C - 50000)/8000$$

$$\mu_2 = (.9 - x_1)/.2$$

$$\mu_3 = (.9 - x_2)/.2$$

$$\mu_4 = (.9 - x_3)/.2$$

$$\mu_5 = (.9 - x_4)/.2$$

Let $\lambda = \min\{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5\}$ i.e. λ is the min operator, which is the intersection of all the membership functions.

The crisp form for the above fuzzy model is obtained as

Max λ

$24000x_1 + 32000x_2 + 17500x_3 + 6000x_4 - 2296\lambda \geq 60000$

subject to

$x_1 \geq 0.25$

$34.0856x_1 + 80x_2 \geq 84.0856$

$29.0457x_1 + 68.172x_2 + 35x_3 \geq 102.2177$

$24.751x_1 + 58.09209x_2 + 29.8249x_3 + 30x_4 \geq 112.66799$

$x_1 + 0.05\,\lambda \leq .95$

$x_2 + 0.05\,\lambda \leq .95$

$x_3 + 0.05\,\lambda \leq .95$

$x_4 + 0.05\,\lambda \leq .95$

$x_1 \geq 0,\ \ x_2 \geq 0,\ \ x_3 \geq 0,\ \ x_4 \geq 0,\ \lambda \geq 0$ …………………… (2.4)

ANALYSIS: Numerical solution to the problem (2.4) has been obtained using LINDO software. The optimal solution is as ;

$\lambda = .551392$

$\lambda = \min\{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5\}$

$$\mu_1 = .55139$$

$$\mu_2 = .56039$$

$$\mu_3 = .56039$$

$$\mu_4 = .56039$$

$$\mu_5 = .56039$$

& total WPC cost is 54411.12 units.

RESULTS/FINDINGS

Based on the above work and it is inferred that total WPC cost is 54411.12 units

DISCUSSION/ANALYSIS

CONCLUSION

This paper is about solving a practical problem with pollution control of water using optimization technique in fuzzy environment known as fuzzy linear programming approach. This approach deals with min operator. It is observed that by allowing slight flexibility or fuzziness to different goals, the objective value is considerably minimized. There is ample scope for further generalization and diversifying the problem in keeping with real life situation and corresponding modifications in the mathematical model developed in this work.

LIMITATION AND STUDY FORWARD

We have use fuzzy programming to solve this problem with is vague in nature. Hence it is not possible to give the accurate date and amount of purified water. Other can use different methods to solve this problem

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