(λ,μ)-Multi Anti Fuzzy subgroup of a group

Kr. Balasubramanian[1], R. Revathy[2*]

[1]Assistant Professor in Mathematics, H.H. The Rajah's college(Affiliated to bharathidasan University, Trichy), Pudukkottai, Tamil Nadu, India; [2*]Research Scholar in Mathematics, H.H .The Rajah's college(Affiliated to bharathidasan University, Trichy), Pudukkottai, Tamil Nadu, India.

Email: [1]balamohitha@gmail.com, [2*]revathyrsmaths@gmail.com

Keywords

(λ,μ)-Multi Anti Fuzzy Set((λ,μ)-MAFS), (λ,μ)-Multi Anti Fuzzy Subgroup ((λ,μ)- MAFSG), (λ,μ)-Multi Anti Fuzzy Normal Subgroup ((λ,μ)-MAFNSG).

Article History

Received on 24th May 2022

Accepted on 6th July 2022

Published on 2nd August 2022

Cite this article

Balasubramanian, K., & Revathy, R. (2022). (λ,μ)-Multi Anti Fuzzy subgroup of a group. *International Journal of Students' Research in Technology & Management*, *10*(3), 25-33. https://doi.org/10.18510/ijsrtm.2022.1035

Abstract

Purpose of the study: To develop (λ, μ) - anti fuzzy subgroup of a group.

Methodology: The fundamental idea of (λ, μ) - anti fuzzy subgroup to create a (λ, μ)- multi anti fuzzy subgroup.

Main Findings: (λ, μ) – multi anti fuzzy cosets of a group.

Applications of this study: The advancement of the theory of a group's multiple fuzzy subgroups.

Novelty/Originality of this study: The concept of (λ, μ) - multi anti fuzzy cosets of a group has been defined, and various associated theorems have been demonstrated using examples.

INTRODUCTION

Fuzzy sets were first introduced by Feng, Y. and Yao, B.(2012) and then the fuzzy sets have been used in the reconsideration of classical mathematics. Yuan, X., Zhang, C., and Ren, Y.(2003) introduced the concept of fuzzy subgroup with thresholds. A fuzzy subgroup with thresholds λ and μ is also called a (λ, μ)-fuzzy subgroup. Yao continued to research (λ, μ)-fuzzy normal subgroups, (λ, μ)-fuzzy quotient subgroups and (λ, μ)-fuzzy subrings in (Yao, B.(2005)). Shen researched anti-fuzzy subgroups in (Shen, Z.(1995)) By a fuzzy subset of a non-empty set X we mean a mapping from X to the unit interval [0,1]. Throughout this article, we will always assume that $0 \le \lambda < \mu \le 1$. (Atanassov, K.T. (1986), Atanassov, K.T. (1994), Mukharjee, N. P. and Bhattacharya, P.(1984), Zadeh, L.A.(1965))

Sabu, S., Ramakrishnan, T.V.(2010), Sabu, S. and Ramakrishnan, T.V.(2011a) and Sabu, S., Ramakrishnan, T.V.(2011b) proposed the theory of multi fuzzy sets in terms of multi dimentional membership functions and investigated some properties of multi level fuzziness. An element of a multi fuzzy set can occur more than once with possibly [same or different membership values]. Muthuraj, R. and Balamurugan, S.(2013) and Muthuraj, R. and Balamurugan, S.(2014) proposed the intuitionistic multi anti fuzzy subgroup and its lower level subgroups. Balasubramanian, K.R., Revathy, R and Rajangam, R.(2021) introduced the notion of (λ, μ)-multi fuzzy set and (λ, μ)-multi fuzzy subgroup of a group. In this paper we study a detailed investigation on (λ, μ)-multi anti fuzzy subgroups of a group. (Basnet, D.K. & Sarma, N.K.(2010), Biswas, R.(2006), Goguen, J.A.(1967), Rosenfeld, A.(1971), Sinoj, T.K. and Sunil, J.J.(2013))

Preliminaries Definition: 2.1 (Feng, Y. and Yao, B.(2012))

Let X be a non-empty set. A fuzzy subset A of X is defined by a function A:X⟶[0,1].

Definition: 2.2 (Sabu, S. and Ramakrishnan, T.V.(2011a), Sabu, S., Ramakrishnan, T.V.(2011b))

Let X be a non-empty set. A multi fuzzy set A in X is defined as the set of ordered sequences as follows. $A = \{(x, A_1(x), A_2(x), \dots, A_k(x), \dots): x \in X\}$. Where $A_i: X \longrightarrow [0,1]\ for\ all\ i$.

Definition: 2.3 (Sabu, S., Ramakrishnan, T.V.(2011b))

Let X be a non-empty set. A k-dimensional multi fuzzy set A in X is defined by the set

$A = \{(x, (A_1(x), A_2(x), \dots A_k(x))), : x \in X\}$. Where $A_i: X \longrightarrow [0,1]\ for\ i = 1,2,3, \dots, k$.

Definition: 2.4 (Feng, Y. and Yao, B.(2012))

Let A be a fuzzy subset of G. A is called a (λ, μ)-anti fuzzy subgroup of G if, for all $x, y \in G$,

$(i)(xy)$ A $\mu \leq (x) \vee A(y) \vee \lambda (ii)\ (x^{-1})$ A $\mu \leq A(x) \vee \lambda$

Clearly, a (0, 1)-anti fuzzy subgroup is just an anti fuzzy subgroup, and thus a (λ, μ)-anti fuzzy subgroup is a generalization of fuzzy subgroup.

MAIN RESULTS

Definition: 3.1

Let A be a fuzzy subset of G. Then a (λ, μ)- anti fuzzy subset $A^{(\lambda,\mu)}$ of a fuzzy set A of G isdefined as, $A^{(\lambda,\mu)} = (x, \{A$ A $(1-\lambda)\} \vee (1-\mu): x \in G)$.

Definition: 3.2

Let A be a multi fuzzy subset of G. Then a (λ, μ)- multi anti fuzzy subset $A^{(\lambda,\mu)}$ of a multifuzzy set A of G is defined as, $A^{(\lambda,\mu)} = (x, \{A$ A $(1-\lambda)\} \vee (1-\mu) : x \in G)$. That is, $^{(\lambda i,\mu i)} = (x, \{A_i$ A $(1-\lambda_i)\} \vee (1-\mu_i) : x \in G)$.

Clearly, a (0, 1)-multi anti fuzzy subset is just a multi fuzzy subset of G, and thus a (λ, μ)- multi anti fuzzy subset is also a generalization of multi fuzzy subset. Where (0,1)-multi anti fuzzy subset A is defined as $A^{(0,1)} = (A_i^{(0_i,1_i)})$.

Definition: 3.3

Let A be a multi fuzzy subset of G. $A = (A_i)$ is called a (λ, μ)-multi anti fuzzy subgroup of G

if, for all $x \in G$, $A(xy) \vee \mu \leq \max\{A(x), A(y)\}$ A λ,

That is,

$A_i(xy) \vee \mu_i \leq max\{A_i(x), \{A_i(y)\}$ A λ_i

Clearly, a (0, 1)-multi anti fuzzy subgroup is just a multi anti fuzzy subgroup of G, and thus a(λ, μ)- multi anti fuzzy subgroup is also a generalization of multi anti fuzzy subgroup.

Definition: 3.4

Let $^{(\lambda,\mu)}$ and $B^{(\lambda,\mu)}$ be any two (λ, μ)- multi anti fuzzy sets having the same dimension k of X.

Then

$(i)\ A^{(\lambda,\mu)} \subseteq B^{(\lambda,\mu)}$, iff $A^{(\lambda,\mu)}(x) \leq B^{(\lambda,\mu)}(x)$ for all $x \in X$

$(ii)\ A^{(\lambda,\mu)} = B^{(\lambda,\mu)}$, iff $A^{(\lambda,\mu)}(x) = B^{(\lambda,\mu)}(x)$ for all $x \in X$

$(iii)\wedge^{(\lambda,\mu)} = \{(x, 1 - A^{(\lambda,\mu)}): x \in X\}$

$(iv)^{(\lambda,\mu)} \cap {}^{(\lambda,\mu)} = \{(x, (A^{(\lambda,\mu)} \cap B^{(\lambda,\mu)})(x): x \epsilon X\}$,

$where\ (A^{(\lambda,\mu)} \cap B^{(\lambda,\mu)})(x) = \min\{A^{(\lambda,\mu)}(x), B^{(\lambda,\mu)}(x)\} = \min\{A_i^{(\lambda i,\mu i)}(x), B_i^{(\lambda i,\mu i)}(x)\}\ \ for\ i = 1,2, \dots, k$

$(v)A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)} = \{(x, A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)}(x)): x \in X\}$,

$where\ (A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)})(x) = \max\{A^{(\lambda,\mu)}(x), B^{(\lambda,\mu)}(x)\} = \max\{A_i^{(\lambda i,\mu i)}(x), B_i^{(\lambda i,\mu i)}(x)\}\ \ for\ i = 1,2, \dots, k$

Here, $\{A_i^{(\lambda i,\mu i)}(x)\}$ and $\{B_i^{(\lambda i,\mu i)}(x)\}$ represents the corresponding ith position membership values of $A^{(\lambda,\mu)}$ and $B^{(\lambda,\mu)}$.

Definition: 3.5

Let $A^{(\lambda,\mu)} = \{(x, A^{(\lambda,\mu)}(x)): x \in X\}$ be a (λ, μ) −MAFS of dimension k and let $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_k) \in [0,1]^k$, where each $\alpha_i \in [0,1]$ for all i. Then the $\alpha - lowerer\ cut$ of $^{(\lambda,\mu)}$ is the set of all x such that $A_i^{(\lambda i,\mu i)}(x) \leq \alpha_i$, $\forall i$ and is denoted by $[A^{(\lambda,\mu)}]_{(\alpha)}$. Clearly it is a crisp set.

Definition: 3.6

Let $A^{(\lambda,\mu)} = \{(x, A^{(\lambda,\mu)}(x)): x \in X\}$ be a (λ, μ) −MAFS of dimension k and let $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_k) \in [0,1]^k$, where each $\alpha_i \in [0,1]$ for all i. Then the $strong\ \ \alpha - lower\ cut$ of

$^{(\lambda,\mu)}$ is the set of all x such that $A_i^{(\lambda,\mu)}(x) < \alpha_i, \forall i$ and is denoted by $[A^{(\lambda,\mu)}]_{\alpha}*$. Clearly it is alsoa crisp set.

Theorem: 3.7 (Feng, Y. and Yao, B.(2012))

Let A and B are any two (λ, μ) −MAFSs of dimension k taken from a non –empty set X. Then

$A \subseteq B$ if and only if $[A^{(\lambda,\mu)}]_{(\alpha)} \subseteq [B^{(\lambda,\mu)}]_{(\alpha)}$for every $\in [0,1]^k$.

Definition: 3.8

A MFS A = {(x, A(x)): x ∈ X} of a group G is said to be a (λ, μ)-multi anti fuzzy sub group of G((λ, μ)-MAFSG), if it satisfies the following: For $\lambda, \mu \in [0,1]^k$, $0 \le \lambda_i \le \mu_i \le 1$, $0 \le \lambda_i + \mu_i \le 1$ (i) $A(xy) \wedge \mu \le \max\{A(x), A(y)\} \vee \lambda$

(ii) $(x^{-1}) \wedge \mu \le (x) \vee \lambda$ for all $x, y \in G$. That is,

(i) $(xy) \wedge \mu_i \le \max\{A_i(x), A_i(y)\} \vee \lambda_i$

(ii) $(x^{-1}) \wedge \mu_i \le A_i(x) \vee \lambda_i$ for all $x, y \in G$.

Clearly, a (0, 1)-multi anti fuzzy subgroup is just a multi anti fuzzy subgroup of G, and thus a (λ, μ)- multi anti fuzzy subgroup is a generalization of multi anti fuzzy subgroup.

(i) If A is a (λ, μ) −MAFSG of G, then the complement of A need not be an (λ, μ) −MAFSG of G.

(ii) A is a MAFSG of a group ⟺ each (λ, μ) −AFS $^{(\lambda_i, \mu_i)}$ is a $(\lambda, \mu) - A$FSG of G. i=1,2,...,k

Definition: 3.10 (Muthuraj, R. and Balamurugan, S.(2013))

A (λ, μ) − MAFSG $^{(\lambda,)}$ of a group G is said to be a (λ, μ) −multi anti fuzzy normal subgroup ($(\lambda, \mu) - MAFNSG$) of G, it satisfies $A^{(\lambda,\mu)}(xy) = A^{(\lambda,\mu)}(yx)$ for all $x, y \in G$

Definition: 3.11

Let $(G, .)$ be a $Groupoid$ and $A^{(\lambda,\mu)}$, $B^{(\lambda,\mu)}$ be any two (λ, μ) −MAFSs having same dimension k of G. Then the product of $^{(\lambda,)}$ and $B^{(\lambda,\mu)}$, denoted by $A^{(\lambda,\mu)} \circ B^{(\lambda,\mu)}$ and is defined as:

$$A^{(\lambda,\mu)} \circ B^{(\lambda,\mu)}(x) = \begin{cases} \max[\min\{A^{(\lambda,\mu)}(y), B^{(\lambda,\mu)}(z)\} : yz = x, \forall\, y, z \in G] \\ \lambda_k = (\lambda, \lambda, \dots, \lambda_{k\ times}), if\ x\ is\ not\ expressible\ sa\ x = yz \end{cases}, \forall\, x \in G$$

That is, $\forall\, x \in G$,

$$A^{(\lambda,\mu)} \circ B^{(\lambda,\mu)}(x) = \begin{cases} (\max[\min\{A^{(\lambda,\mu)}(y), B^{(\lambda,\mu)}(z)\} : yz = x, \forall\, y, z \in G] \\ (\lambda_k),\ if\ x\ is\ not\ expressible\ as\ x = yz \end{cases}$$

Definition: 3.12

Let X and Y be any two non-empty sets and $f: X \longrightarrow Y$ be a

mapping. Let $^{(\lambda,)}$ and $B^{(\lambda,\mu)}$ be any two (λ, μ) −MAFSs of X and Y respectively having the same dimension k. Then the image of $^{(\lambda,)}(\subseteq X)$ under the map f is denoted by $f(A^{(\lambda,\mu)})$, is defined as: $\forall y \in Y$,

$$f(A^{(\lambda,\mu)})(y) = \begin{cases} \max\{A^{(\lambda,\mu)}(x) : x \in f^{-1}(y) \\ \lambda_k, & otherwise \end{cases}$$

Also, the $pre - image$ of $B^{(\lambda,\mu)}(\subseteq Y)$ under the map f is denoted by $f^{-1}(B^{(\lambda,\mu)})$ and it isdefined as: $f^{-1}(B^{(\lambda,\mu)})(x) = (B^{(\lambda,\mu)}(f(x)), \forall x \in X$.

Properties of (α, Q) −lower cuts of the (λ, μ) −MAFSG's of a group

In this section, we have proved some theorems on (λ, μ)- IMAFSG's of a group G by using some of their $(\alpha, \beta) - Lower\ cuts$.

Proposition: 4.1

If $^{(\lambda,)}$ and $B^{(\lambda,\mu)}$ are any two (λ, μ)-MAFSs of a universal set X

Then the following are holds good :

(i) $[A^{(\lambda,\mu)}]_\alpha \subseteq [^{(\lambda,)}]_\delta$ if $\alpha \le \delta$

(ii) $A^{(\lambda,\mu)} \subseteq B^{(\lambda,\mu)}$ implies $[B^{(\lambda,\mu)}]_\alpha \subseteq [A^{(\lambda,\mu)}]_\alpha$

(iii) $[A^{(\lambda,\mu)} \cap B^{(\lambda,\mu)}]_{\bar{\bar{\alpha}}}$ $[A^{(\lambda,\mu)}]_\alpha \cap [B^{(\lambda,\mu)}]_\alpha$

(iv) $[A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)}] \subseteq [A^{(\lambda,\mu)}]_\alpha \cup [B^{(\lambda,\mu)}]_\alpha$ $(here\ equality\ holds\ if\ \alpha_i = 1, \forall\, i)$$(v)$ $[\cap A_i^{(\lambda_i,\mu_i)}]_\alpha = \cap [A_i^{(\lambda_i,\mu_i)}]_\alpha$, $where\ \alpha, \delta \in [0,1]^k$

Proposition: 4.2

Let (G,.) be a groupoid and $A^{(\lambda,\mu)}$ and $B^{(\lambda,\mu)}$ are any two (λ,μ) −MAFS's of G. Then we have

$[A^{(\lambda,\mu)} \circ B^{(\lambda,\mu)}]_{\alpha} = [A^{(\lambda,\mu)}]_{\alpha} [B^{(\lambda,\mu)}]_{\alpha}$, where $\alpha \in [0,1]^k$.

Theorem: 4.3

If $^{(\lambda,)}$ is a (λ,μ)-multi anti fuzzy subgroup of G and $\alpha \in [0,1]^k$, then the $\alpha - lower\ cut$ of $^{(\lambda,)}$, $[A^{(\lambda,\mu)}]$ is a subgroup of G, where $A^{(\lambda,\mu)}(e) \leq \alpha$ and 'e' is the identity element of G.

Proof:

We have, $^{(\lambda,)}(e) \leq \alpha$,$e \in [A^{(\lambda,\mu)}]_{\alpha}$. Therefore $[^{(\lambda,)}]_{\alpha} \neq \emptyset$.

Let $x, y \in [^{(\lambda,)}]_{\alpha}$. Then $^{(\lambda,)}(x) \leq \alpha\ and\ A^{(\lambda,\mu)}(y) \leq \alpha$.

Then for all i, $^{(\lambda i,\mu i)}(x) \leq \alpha_i$ and $A_i^{(\lambda i,\mu i)}(y) \leq \alpha_i$.

$\Longrightarrow max\{A_i^{(\lambda i,\mu i)}(x), A_i^{(\lambda i,\mu i)}(y)\} \leq \alpha_i, \forall\ i \ldots \ldots \ldots \ldots (1)$

$\Longrightarrow A_i^{(\lambda i,\mu i)}(xy^{-1}) \leq max\{A_i^{(\lambda i,\mu i)}(x), A_i^{(\lambda i,\mu i)}(y)\} \leq \alpha_i$,$\forall\ i$

since $A^{(\lambda,\mu)}$ is a (λ,μ)-multi anti fuzzy subgroup of a group G and by (1).

$\Longrightarrow\ ^{(\lambda i,\mu i)}(xy^{-1}) \leq \alpha_i$, $\forall\ i$

$\Longrightarrow\ ^{(\lambda,)}(xy^{-1}) \leq \alpha$

$\Longrightarrow xy^{-1} \in [^{(\lambda,)}]_{\alpha}$

$\Longrightarrow [^{(\lambda,)}]_{\alpha}$ is a subgroup of G.

Theorem: 4.4

If $A^{(\lambda,\mu)}$ is a (λ,μ)- multi anti fuzzy subset of a group G, then $A^{(\lambda,\mu)}$ is a (λ,μ)-multi anti fuzzy subgroup of $G \Leftrightarrow$ each α − Lower cut $[A^{(\lambda,\mu)}]_{\alpha}$ is a subgroup of G, for all $\alpha \in [0,1]^k$ with $0 \leq \alpha_i \leq 1, \forall\ i$.

Proof:

($\Longrightarrow$)Let $^{(\lambda,)}$ be a (λ,μ) −multi anti fuzzy subgroup of a group G. Then by the above definition:3.6, each $\alpha - Lower\ cut$ $[A^{(\lambda,\mu)}]_{\alpha}$ is a subgroup of G for all $\alpha \in [0,1]^k$ with $0 \leq \alpha_i \leq 1, \forall\ i$.

($\Longleftarrow$) Conversely, let $A^{(\lambda,\mu)}$ be a (λ,μ)- multi anti fuzzy subset of a group G such that each $\alpha - Lower\ cut$ $[A^{(\lambda,\mu)}]_{\alpha}$ is a subgroup of G for all $\alpha, \beta \in [0,1]^k$ with $0 \leq \alpha_i \leq 1, \forall\ i$.

To prove that $^{(\lambda,)}$ is a (λ,μ)-multi anti fuzzy subgroup of G. we must prove :

$(i)\ A^{(\lambda,\mu)}(xy) \leq max\{A^{(\lambda,\mu)}(x), A^{(\lambda,\mu)}(y)\}\ \forall\ x, y \in G (ii)\ A^{(\lambda,\mu)}(x^{-1}) = A^{(\lambda,\mu)}(x)$

Let $x, y \in G$ and for all i , let $\alpha_i = \max\{A_i^{(\lambda i,\mu i)}(x), A_i^{(\lambda i,\mu i)}(y)\}$.Then $\forall\ i$,

We have $^{(\lambda i,\mu i)}(x) \leq \alpha_i$, $A_i^{(\lambda i,\mu i)}(y) \leq \alpha_i$

That is, $\forall\ i$,we have $A_i^{(\lambda i,\mu i)}(x) \leq \alpha_i$ and $A_i^{(\lambda i,\mu i)}(y) \leq \alpha_i$,Then we have $A^{(\lambda,\mu)}(x) \leq \alpha$ and $A^{(\lambda,\mu)}(y) \leq \alpha$

That is, $x \in [A^{(\lambda,\mu)}]_{\alpha}$ and $y \in [A^{(\lambda,\mu)}]_{\alpha}$ therefore, $xy \in [A^{(\lambda,\mu)}]_{\alpha}$, since each $[A^{(\lambda,\mu)}]_{\alpha}$ is asubgroup by hypothesis .

Therefore, $\forall\ i$, we have $A_i^{(\lambda i,\mu i)}(xy) \leq \alpha_i = \max\{A_i^{(\lambda i,\mu i)}(x), A_i^{(\lambda i,\mu i)}(y)\}$. ie., $A^{(\lambda,\mu)}(xy) \leq max\ \{A^{(\lambda,\mu)}(x), A^{(\lambda,\mu)}(y)\}$. Hence (i) is true.

Now, let $x \in G\ and\ \forall\ i$, let $^{(\lambda i,\mu i)}(x) = \alpha_i$. Then $^{(\lambda i,\mu i)}(x) \leq \alpha_i$ is true for all i.

Therefore, $^{(\lambda,)}(x) \leq \alpha$. Thus , $x \in [A^{(\lambda,\mu)}]_{\alpha}$.

Since each $[A^{(\lambda,\mu)}]_{\alpha}$ is a subgroup of G for all $\alpha \in [0,1]^k$ and $x \in [A^{(\lambda,\mu)}]_{\alpha}$, we have $x^{-1} \in [A^{(\lambda,\mu)}]_{\alpha}$ which implies that $A_i^{(\lambda i,\mu i)}(x^{-1}) \leq \alpha_i$ is true, $\forall\ i$. Which implies that $^{(\lambda i,\mu i)}(x^{-1}) \leq A_i^{(\lambda i,\mu i)}(x)$ is true, $\forall\ i$. Thus,$\forall\ i$, $A_i^{(\lambda i,\mu i)}(x) = A_i^{(\lambda i,\mu i)}((x^{-1})^{-1}) \leq A_i^{(\lambda i,\mu i)}(x^{-1}) \leq A_i^{(\lambda i,\mu i)}(x)$, which implies that $A_i^{(\lambda i,\mu i)}(x^{-1}) = A_i^{(\lambda i,\mu i)}(x)$. Hence $^{(\lambda,)}$ is a (λ,μ) −multi antifuzzy subgroup of G.

Theorem: 4.5

If $A^{(\lambda,\mu)}$ is an (λ,μ)- multi anti fuzzy normal subgroup of a group G and for every , $\alpha \in [0,1]^k$, then the $\alpha - Lower\ cut$ $[A^{(\lambda,\mu)}]_{\alpha}$ is a normal subgroup of G, where $A^{(\lambda,\mu)}(e) \leq \alpha$ and 'e' is the identity element of G.

Proof:

Let $x \in [^{(\lambda,)}]_\alpha$ and $g \in G$. Then $^{(\lambda,)}(e) \leq \alpha$.

That is, $^{(\lambda_i,\mu_i)}(x) \leq \alpha_i \quad , \forall\, i$ .... (1)

Since $^{(\lambda,)}$ is a (λ,μ)-MAFNSG of G,

$^{(\lambda_i,\mu_i)}(g^{-1}xg) \;=\; A_i^{(\lambda_i,\mu_i)}(x)\;,\; \forall\; i.$

$\Longrightarrow\; ^{(\lambda_i,\mu_i)}(g^{-1}xg) = A_i^{(\lambda_i,\mu_i)}(x) \leq \alpha_i$ and $\forall\, i$,by using (1).

$\Longrightarrow\; ^{(\lambda_i,\mu_i)}(g^{-1}xg) \leq \alpha_i,\; \forall\; i$

$\Longrightarrow\; ^{(\lambda,)}(g^{-1}xg) \leq \alpha \Longrightarrow g^{-1}xg \in [A^{(\lambda,\mu)}]_\alpha$

$\Longrightarrow [^{(\lambda,)}]_\alpha$ is normal subgroup of G.

Theorem: 4.6

If $A^{(\lambda,\mu)}$ and $B^{(\lambda,\mu)}$ are any two (λ,μ)- multi anti fuzzy subgroups ((λ,μ)-MAFSGs) of a group G, then $(A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)})$ is also an (λ,μ)- multi anti fuzzy subgroup of G.

Proof:

$^{(\lambda,\mu)}(x^{-1}) \;=\; A^{(\lambda,\mu)}(x)$

Assume $^{(\lambda,)}$ and $B^{(\lambda,\mu)}$ are any two (λ,μ)-multi anti fuzzy subgroup of a group G, then $\forall\, x, y \in G$,

(i) $^{(\lambda,)}\;(xy^{-1}) \leq \max\{A^{(\lambda,\mu)}\;(x), A^{(\lambda,\mu)}\;(y)\}$ and

(ii) $B^{(\lambda,\mu)}\;(xy^{-1}) \leq \max\{B^{(\lambda,\mu)}\;(x), B^{(\lambda,\mu)}\;(y)\}$ ……(1) Now, $(A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)})(x) = \max\{A^{(\lambda,\mu)}(x), B^{(\lambda,\mu)}(x)\}$.

Then $(A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)})(xy^{-1}) = \max\{A^{(\lambda,\mu)}(xy^{-1}), B^{(\lambda,\mu)}(xy^{-1})\} \leq \max\{\max\{A^{(\lambda,\mu)}(x), A^{(\lambda,\mu)}(y)\}\;,\; \max\{B^{(\lambda,\mu)}(x), B^{(\lambda,\mu)}(y)\}\}\;, by\;(1)$

$=\max\{\max\{A^{(\lambda,\mu)}(x), B^{(\lambda,\mu)}(x)\}\;,\; \max\{A^{(\lambda,\mu)}(y), B^{(\lambda,\mu)}(y)\}\}$

$=\max\{(A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)})(x), (A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)})(y)\}\;,\; by\;(1).$

That is, $(A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)})(xy^{-1}) \leq \max\{(A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)})(x), (A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)})(y)\}$ for all x, y in G. Hence $(^{(\lambda,)} \cup B^{(\lambda,\mu)})$ is a (λ,μ) −multi antifuzzy subgroup of G.

Remark: 4.7

The intersection of two (λ,μ) −multi antifuzzy subgroups of a group G need not be a (λ,μ)-MAFSG of the group G.

Proof:

Consider the Klein's four group G={e, a, b, ab }, where $a^2 = e = b^2$ and $ba = ab$. For $0 \leq i \leq 5$, $let\; r_i, s_i \in [0,1]^k$ such that $r_0 < r_1 <\ldots. < r_5$ and $s_0 > s_1 >\ldots. > s_5$. Define $(\lambda,\mu) - MAFSs\; A^{(\lambda,\mu)}$ and $B^{(\lambda,\mu)}$ of dimension k as follows : $A^{(\lambda,\mu)} = \{(x, A^{(\lambda,\mu)}) : x \in G\}$ and $B^{(\lambda,\mu)} = \{(x, B^{(\lambda,\mu)}) : x \in G\}$, where $A_i^{(\lambda_i,\mu_i)}(e) = r_1 \wedge (1-\lambda_i) \vee (1-\mu_i)$, $A_i^{(\lambda_i,\mu_i)}(a) = r_3 \wedge (1-\lambda_i) \vee (1-\mu_i)$, $A_i^{(\lambda_i,\mu_i)}(b) = r_4 \wedge (1-\lambda_i) \vee (1-\mu_i) = A_i^{(\lambda_i,\mu_i)}(ab)$ and $B_i^{(\lambda_i,\mu_i)}(e) = r_0 \wedge (1-\lambda_i) \vee (1-\mu_i)$, $B_i^{(\lambda_i,\mu_i)}(a) = r_5 \wedge (1-\lambda_i) \vee (1-\mu_i) = B_i^{(\lambda_i,\mu_i)}(ab)$, $B_i^{(\lambda_i,\mu_i)}(b) = r_2 \wedge (1-\lambda_i) \vee (1-\mu_i)$.

Clearly $^{(\lambda,)}$ and $B^{(\lambda,\mu)}$ are (λ,μ)-multi anti fuzzy subgroups of G.

Now $A^{(\lambda,\mu)} \cap B^{(\lambda,\mu)} = \{A_i^{(\lambda_i,\mu_i)} \cap B_i^{(\lambda_i,\mu_i)}\} = \{(x, (A_i^{(\lambda_i,\mu_i)} \cap B_i^{(\lambda_i,\mu_i)})(x)) : x \in G\}$, where

$(A_i^{(\lambda_i,\mu_i)} \cap B_i^{(\lambda_i,\mu_i)})(x) = \min\{A_i^{(\lambda_i,\mu_i)}(x), B_i^{(\lambda_i,\mu_i)}(x)\} = (\min\{A_i^{(\lambda_i,\mu_i)}(x), B_i^{(\lambda_i,\mu_i)}(x)\})^k_{i=1}$

$(A_i^{(\lambda_i,\mu_i)} \cap B_i^{(\lambda_i,\mu_i)})(e) = r_0 \wedge (1-\lambda_i) \vee (1-\mu_i)$, $(A_i^{(\lambda_i,\mu_i)} \cap B_i^{(\lambda_i,\mu_i)})(a) = r_3 \wedge$
$(1-\lambda_i) \vee (1-\mu_i)$, $(A_i^{(\lambda_i,\mu_i)} \cap B_i^{(\lambda_i,\mu_i)})(b) = r_2 \wedge (1-\lambda_i) \vee (1-\mu_i)$; $A_i^{(\lambda_i,\mu_i)}(ab) = r_4 \wedge (1-\lambda_i) \vee (1-\mu_i)$;

$[A_i^{(\lambda_i,\mu_i)}]_{(r_3,s_4)} = \{x : x \in G\; such\; that\; ^{(\lambda_i,\mu_i)}(x) \leq r_3\} = \{e, a\}$

$[B_i^{(\lambda_i,\mu_i)}]_{(r_3,s_4)} = \{x : x \in G\; such\; that\; ^{(\lambda_i,\mu_i)}(x) \leq r_3\;\} = \{e\;\}$

$[A_i^{(\lambda_i,\mu_i)} \cap B_i^{\;(\lambda_i,\mu_i)}]_{(t_3,s_4)} = \{x : x \in G\; such\; that\; (^{(\lambda_i,\mu_i)} \cap B_i^{\;(\lambda_i,\mu_i)})(x) \;\leq r_3\;\}$

$= \{x : x \in G\; such\; that\; \min\{A_i^{(\lambda_i,\mu_i)}(x), B_i^{(\lambda_i,\mu_i)}(x)\} \leq r_3\} = \{e, a, b\}$

Since {e, a, b } is not a subgroup of G, $[A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)}]_{(r_3\ s_4)}$ is not a subgroup of G. Hence $[^{(\lambda,)} \cup B^{(\lambda,\mu)}]$ is not a subgroup of G and there fore $[A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)}]$ is not a (λ, μ)-MAFSG of the group G.

Example: 4.8

There are two cases needed to clarify the previous theorem 3.7 and remark.

$Case\ (i)$: Consider the abelian group $G = \{e, a, b, \}$ with usual multiplication such that $a^2 = e = b^2$ and $ab = ba$. Let $A^{(\lambda,\mu)} = \{< e, (0.3 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.2 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < a, (0.5 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.6 \wedge (1-\lambda_2) \vee (1-\mu_2) >, < b, (0.5 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.6 \wedge (1-\lambda_2) \vee (1-\mu_2)), >, < ab, (0.6 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.6 \wedge (1-\lambda_2) \vee (1-\mu_2)) >\}$ and $B^t = \{< e, (0.2 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.3 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < a, (0.7 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.8 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < b, (0.4 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.6 \wedge (1-\lambda_2) \vee (1-\mu_2)), < ab, (0.7 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.8 \wedge (1-\lambda_2) \vee (1-\mu_2)) >\}$ be two (λ, μ)-MAFSs having dimension two of G. Clearly $^{(\lambda,)}$ and $B^{(\lambda,\mu)}$ are (λ, μ) $-MAFSGs$ of G.

Then $A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)} = \{< e, (0.3 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.3 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, <a, (0.7 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.8 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < b, (0.6 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.6 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < ab, (0.7 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.8 \wedge (1-\lambda_2) \vee (1-\mu_2)) >\}$ and $A^{(\lambda,\mu)} \cap B^{(\lambda,\mu)} = \{< e, (0.2 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.2 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < a, (0.5 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.6 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < b, (0.4 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.6 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < ab, (0.6 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.6 \wedge (1-\lambda_2) \vee (1-\mu_2)) > \}$

Therefore it is easily verified that in this case $^{(\lambda,)} \cup B^{(\lambda,\mu)}$ is a $(\lambda, \mu) - MAFSG$ of G and $A^{(\lambda,\mu)} \cap B^{(\lambda,\mu)}$ is not a $(\lambda, \mu) - MAFSG$ of G. Hence $ca(i)$.

$Ca(ii)$: Consider the abelian group $G = \{e, a, b, ab \}$ with usual multiplication such that $a^2 = e = b^2$ and $ab = ba$. Let $A^{(\lambda,\mu)} = \{< e, (0 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.1 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < a, (0 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.4 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < b, (0 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.8 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < ab, (0 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.8 \wedge (1-\lambda_2) \vee (1-\mu_2)) >\}$ and $B^{(\lambda,\mu)} = \{< e, (0.3 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.3 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, <a, (0.8 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.6 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < b, (0.8 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.9 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < ab, (0.6 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.9 \wedge (1-\lambda_2) \vee (1-\mu_2)) >\}$ be two (λ, μ)-MAFSs having dimension two of G. Clearly $^{(\lambda,)}$and $B^{(\lambda,\mu)}$are (λ, μ) $-MAFSGs$ of G.

Then $A^{(\lambda,\mu)} \cup B^{(\lambda,\mu)} = \{< e, (0.3 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.3 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, <a, (0.8 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.6 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < b, (0.8 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.9 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < ab, (0.6 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.9 \wedge (1-\lambda_2) \vee (1-\mu_2)) >\}$ and $A^{(\lambda,\mu)} \cap B^{(\lambda,\mu)} = \{< e, (0 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.1 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < a, (0 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.4 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < b, (0 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.8 \wedge (1-\lambda_2) \vee (1-\mu_2)) >, < ab, (0 \wedge (1-\lambda_1) \vee (1-\mu_1), 0.8 \wedge (1-\lambda_2) \vee (1-\mu_2)) > \}$.

Here, it can be easily verified that both $^{(\lambda,)} \cup B^{(\lambda,\mu)}$ and $A^{(\lambda,\mu)} \cap B^{(\lambda,\mu)}$ are (λ, μ) $-MAFSGs$ of G. Hence $case\ (ii)$.

From the conclusion of the above example, we come to the point that there is an uncertainty inverifying whether or not $A^{(\lambda,\mu)} \cap B^{(\lambda,\mu)}$ is a $(\lambda, \mu) - MAFSG$ of G.

(λ, μ)-multi anti fuzzy cosets of a group Definition: 5.1

Let G be a group and $^{(\lambda,)}$ be a $(\lambda, \mu) - MAFSG$ of G. Let $x \in G$ be a fixed element. Then the set $A^{(\lambda,\mu)}(g) = A^{(\lambda,\mu)}(x^{-1}g)$,$\forall\ g \in G$ is called the (λ, μ)- multi anti fuzzy left coset of G determined by $A^{(\lambda,\mu)}$ and x.

Similarly ,the set $A^{(\lambda,\mu)}(g) = A^{(\lambda,\mu)}(gx^{-1})$,$\forall g \in G$ is called the (λ, μ)-multi anti fuzzy right coset of G determined by $A^{(\lambda,\mu)}$ and x.

Remark: 5.2

It is clear that if $^{(\lambda,)}$ is a (λ, μ)-multi anti fuzzy normal subgroup of G, then the (λ, μ)-multi anti fuzzy left coset and the (λ, μ)-multi anti fuzzy right coset of $A^{(\lambda,\mu)}$ on G coincides and in this case, we simply call it as (λ, μ)-multi anti fuzzy coset.

Example: 5.3

Let G be a group. Then $A^{(\lambda,\mu)} = \{(x, A^{(\lambda,\mu)}(x)) : x \in G / A^{(\lambda,\mu)}(x) = A^{(\lambda,\mu)}(e)\}$ is a (λ, μ)- multi anti fuzzy normal subgroup of G.

Theorem: 5.4

Let $^{(\lambda,)}$ be a (λ, μ)- multi anti fuzzy subgroup of G and x be any fixed element of G. Then the following holds :

(i) $x[A^{(\lambda,\mu)}]_\alpha = [x\ A^{(\lambda,\mu)}]_\alpha$

(ii) $[A^{(\lambda,\mu)}]_\alpha x = [\ ^{(\lambda,\mu)}x]_\alpha$, $\forall\ \alpha \in [0,1]^k$ with $0 \le \alpha_i \le 1, \forall i$.

Proof:

(i) $[x\ A^{(\lambda,\mu)}]_\alpha = \{g \in G : x\ ^{(\lambda,)}(g) \le \alpha\ \}\ with\ 0 \le \alpha_i \le 1, \forall i$.Also $x[A^{(\lambda,\mu)}]_\alpha = x\{y \in G : A^{(\lambda,\mu)}(y) \le \alpha\ \}= \{xy \in G : A^{(\lambda,\mu)}(y) \le \alpha\}$ … … … … … … … … … … … . (1)

Put $xy = g \Rightarrow y = x^{-1}g$. Then (1) can be written as,

$x[A^{(\lambda,\mu)}]_\alpha = \{g \in G : A^{(\lambda,\mu)}(x^{-1}g) \le \alpha\ \} = \{g \in G : xA^{(\lambda,\mu)}(g) \le \alpha\} = [xA^{(\lambda,\mu)}]_\alpha$

Therefore, $[^{(\lambda,\mu)}]_\alpha = [xA^{(\lambda,\mu)}]_\alpha$, $\forall\ \alpha \in [0,1]^k$ with $0 \le \alpha_i \le 1, \forall i$.

(ii) Now $[A^{(\lambda,\mu)}x]_\alpha = \{g \in G : A^{(\lambda,\mu)}x(g) \le \alpha\ \}\ with\ 0 \le \alpha_i \le 1, \forall i\}$.Also$[A^{(\lambda,\mu)}x]_\alpha x = \{y \in G : A^{(\lambda,\mu)}(y) \le \alpha \ge \beta\}x$

$= \{yx \in G : A^{(\lambda,\mu)}x(y) \le \alpha\ \}$ … … … … … … … … … … … . (2)

Set $yx = g\ \Rightarrow y = gx^{-1}$. Then (2) can be written as $[A^{(\lambda,\mu)}]_\alpha x = \{g \in G : A^{(\lambda,\mu)}(gx^{-1}) \le \alpha\ \}= \{g \in G : A^{(\lambda,\mu)}x(g) \ge \alpha\ \} = [A^{(\lambda,\mu)}x]_\alpha$

Therefore, $[A^{(\lambda,\mu)}]_\alpha x = [A^{(\lambda,\mu)}x]_\alpha$, $\forall\ \alpha \in [0,1]^k$ with $0 \le \alpha_i \le 1, \forall i$.

Homomorphisms of (λ, μ) −Multi fuzzy subgroup

In this section, we shall prove some theorems on (λ, μ) −MAFSG's of a group byhomomorphism.

Preposition: 6.1

Let $f: X \to Y$ be an onto map.If$^{(\lambda,\mu)}$ and $B^{(\lambda,\mu)}$ are two (λ, μ) −multi anti fuzzy sets of multifuzzy sets A and B with dimension k of X and Y respecively , then the following hold:

(i) $f\ ([A^{(\lambda,\mu)}]_{\alpha}) \subseteq [f(A^{(\lambda,\mu)})]_\alpha)$

(ii) $f^{-1}([B^{(\lambda,\mu)}]_\alpha) = [f^{-1}(B^{(\lambda,\mu)})]_\alpha]$, $\forall\ \alpha \in [0,1]^k$ with $0 \le \alpha_i \le 1, \forall i$.

Proof: (i) Let $y \in f\ ([^{(\lambda,)}]_{\alpha})$. Then there exist an element $x \in [^{(\lambda,)}]_{\alpha}$ such that $f(x) = y$. Then we have $^{(\lambda,)}(x) \le \alpha$,

Since $x \in [^{(\lambda,)}]_\alpha$

$\Rightarrow A_i^{(\lambda_i,\mu_i)}(x) \le \alpha_i$

$\Rightarrow \min\{A_i^{(\lambda_i,\mu_i)}(x): x \in f^{-1}\ (y)\} \le \alpha_i$, $\forall i$.

$\Rightarrow \min\{A^{(\lambda,\mu)}(x): x \in f^{-1}\ (y)\} \le \alpha$

$\Longrightarrow f(A^{(\lambda,\mu)})(y) \le \alpha \Longrightarrow y \in [f(A)]_\alpha$

Therefore, $([A^{(\lambda,\mu)}]_\alpha\ ) \subseteq [f(A)]_\alpha, \forall\ A^{(\lambda,\mu)} \in (\lambda, \mu) - MAFS(X)$.

(ii) Let $x \in [f^{-1}(B^{(\lambda,\mu)})]_\alpha \Leftrightarrow \{x \in X : f^{-1}(B^{(\lambda,\mu)})(x) \le \alpha$

- $\{x \in X : f^{-1}(B_i^{(\lambda_i,\mu_i)})(x) \le \alpha_i\}$, $\forall i$.

- $\{x \in X : B_i^{(\lambda_i,\mu_i)}(f(x)) \le \alpha_i\}$, $\forall i$.

- $\{x \in X : B^{(\lambda,\mu)}(f(x)) \le \alpha\ \}$, $\forall i$.

– $\{x \in X : (x) \in [^{(\lambda,\mu)}]_\alpha \Longleftrightarrow \{x \in X : x \in f^{-1}([B^{(\lambda,\mu)}]_\alpha)\}$

- $f^{-1}([B^{(\lambda,\mu)}]_{\alpha})$

Theorem: 6 .2

Let $f: G_1 \to G_2$ be an onto homomorphism and if $^{(\lambda,)}$ is a (λ, μ) −MAFSG of G1, then

$(^{(\lambda,\mu)})$ is a (λ, μ) −MAFSG of group G2.

Proof:

By theorem 4.4, it is enough to prove that each $(\alpha, \beta) - lower\ cuts$ $[(^{(\lambda,\mu)})]_\alpha$ is a subgroupof G_2.$\forall\ \alpha \in [0,1]^k$ with $0 \le \alpha_i \le 1, \forall i$. Let $y_1, y_2 \in [(^{(\lambda,\mu)})]_\alpha$.

Then $(^{(\lambda,\mu)})(y_1) \le \alpha\ and\ f(A^{(\lambda,\mu)})(y_2) \le \alpha$

$\Rightarrow (A^{(\lambda_i,\mu_i)})(y_1) \le \alpha_i\ and\ f(A_i^{(\lambda_i,\mu_i)})(y_2) \le \alpha_i, \forall i$ … … … (1)

By the proposition 6.1(i), we have $f([A^{(\lambda,\mu)}]_\alpha) \subseteq [f(A^{(\lambda,\mu)})]_\alpha), \forall A^{(\lambda,\mu)} \in (\lambda,\mu) - MAFS(G_1)$.

Since f is onto, there exists some x1 and x2 in G1 such that f(x1)=y1 and f(x2)=y2. Therefore, (1)can be written as $f(A_i^{(\lambda_i,\mu_i)})(f(x_1)) \le \alpha_i$ and $f(A_i^{(\lambda_i,\mu_i)})(f(x_2)) \le \alpha_i, \forall i$.

$\Rightarrow A_i^{(\lambda_i,\mu_i)}(x_1) \le f(A_i^{(\lambda_i,\mu_i)})(f(x_1)) \le \alpha_i$ and $A_i^{(\lambda_i,\mu_i)}(x_2) \le f(A_i^{(\lambda_i,\mu_i)})(f(x_2)) \le \alpha_i, \forall i$.

$\Rightarrow {}^{(\lambda_i,\mu_i)}(x_1) \le \alpha_i$ and $A_i^{(\lambda_i,\mu_i)}(x_2) \le \alpha_i, \forall i$.

$\Rightarrow {}^{(\lambda,)}(x_1) \le \alpha$ and $A^{(\lambda,\mu)}(x_2) \le \alpha$,

$\Rightarrow \max\{A^{(\lambda,\mu)}(x_1), A^{(\lambda,\mu)}(x_2)\} \le \alpha$.

$\Rightarrow A^{(\lambda,\mu)}(x_1 x_2^{-1}) \le \max\{A^{(\lambda,\mu)}(x_1), A^{(\lambda,\mu)}(x_2)\}$, since $A^{(\lambda,\mu)} \in (\lambda,\mu) - MAFSG(G_1)$.

$\Rightarrow {}^{(\lambda,)}(x_1 x_2^{-1}) \le \alpha$

$\Rightarrow x_1 x_2^{-1} \in [A^{(\lambda,\mu)}]_\alpha \Longrightarrow f(x_1 x_2^{-1}) \in f([A^{(\lambda,\mu)}]_\alpha) \subseteq [f(A^{(\lambda,\mu)})]_\alpha$

$\Longrightarrow f(x_1)f(x_2^{-1}) \in [f(A^{(\lambda,\mu)})]_\alpha \Longrightarrow f(x_1)f(x_2)^{-1} \in [f(A^{(\lambda,\mu)})]_\alpha \Longrightarrow y_1 y_2^{-1} \in [f(A^{(\lambda,\mu)})]_\alpha$

$\Longrightarrow [f(A^{(\lambda,\mu)})]_\alpha$ is a subgroup of $G_2, \forall \alpha \in [0,1]^k \Rightarrow f(A^{(\lambda,\mu)}) \in (\lambda,\mu) - MAFSG(G_2)$

Corollary: 6.3

If $f: G_1 \to G_2$ be a homomorphism of a group G_1 onto a group G_2 and $\{ {}^{(\lambda_i,\mu_i)} : i \in I \}$ be afamily of $(\lambda,\mu) - MAFSG$s of G_1, then $f(\cup A_i^{(\lambda_i,\mu_i)})$ is an $(\lambda,\mu) - MAFSG$ of G_2.

Theorem: 6.4

Let $f: G_1 \to G_2$ be a homomorphism of a group G_1 into a group G_2. If ${}^{(\lambda,)}$ is an $(\lambda,\mu) - MAFSG$ of G_2, then $f^{-1}(B^{(\lambda,\mu)})$ is also a $(\lambda,\mu) - MAFSG$ of G_1.

Proof:

By theorem 4.4, it is enough to prove that $[f^{-1}(B^{(\lambda,\mu)})]_\alpha$ is a subgroup of G_1, with $0 \le \alpha_i \le 1, \forall i$.

Let $x_1, x_2 \in [f^{-1}(B^{(\lambda,\mu)})]_\alpha$. Then $f^{-1}(B^{(\lambda,\mu)})(x_1) \le \alpha$ and $f^{-1}(B^{(\lambda,\mu)})(x_2) \le \alpha \Longrightarrow B^{(\lambda,\mu)}(f(x_1)) \le \alpha$ and $B^{(\lambda,\mu)}(f(x_2)) \le \alpha$

$\Longrightarrow \max\{B^{(\lambda,\mu)}(f(x_1)), B^{(\lambda,\mu)}(f(x_2))\} \le \alpha$

$\Longrightarrow B^{(\lambda,\mu)}(f(x_1)f(x_2)^{-1} \le \max\{B^{(\lambda,\mu)}(f(x_1)), B^{(\lambda,\mu)}(f(x_2))\} \le \alpha$, since $B^{(\lambda,\mu)} \in (\lambda,\mu) - MAFSG(G_2)$.

$\Longrightarrow (f(x_1)f(x_2)^{-1} \in [B^{(\lambda,\mu)}]_\alpha \Longrightarrow f(x_1 x_2^{-1}) \in [B^{(\lambda,\mu)}]_\alpha$, since f is homomorphism.

$\Longrightarrow x_1 x_2^{-1} \in f^{-1}([B^{(\lambda,\mu)}]_\alpha) = [f^{-1}(B^{(\lambda,\mu)})]_\alpha$, by the preposition 6.1(ii).

$\Longrightarrow x_1 x_2^{-1} \in [f^{-1}(B^{(\lambda,\mu)})]_\alpha \Longrightarrow [f^{-1}(B^{(\lambda,\mu)})]_\alpha$ is a subgroup of G_1.

$\Longrightarrow f^{-1}(B^{(\lambda,\mu)})$ is a $(\lambda,\mu) - MAFSG$ of G_1.

Theorem: 6.5

Let $f: G_1 \to G_2$ be a surjective homomorphism and if ${}^{(\lambda,)}$ is a $(\lambda,\mu) - MAFSG$ of a group G_1,then $f(A^{(\lambda,\mu)})$ is also a $(\lambda,\mu) - MAFNSG$ of a group G_2.

Proof:

Let $g_2 \in G_2$ and $y \in ({}^{(\lambda,\mu)})$. Since f is surjective, there exists $g_1 \in G_1$ and $x \in {}^{(\lambda,)}$,such that $f(x) = y$ and $f(g_1) = g_2$.

Also, since $A^{(\lambda,\mu)}$ is a $(\lambda,\mu) - MAFNSG$ of G_1, $A^{(\lambda,\mu)}(g_1^{-1} x g_1) = A^{(\lambda,\mu)}(x)$,$\forall x \in A^{(\lambda,\mu)}$ and $g_1 \in G_1$.

Now consider, $f(A^{(\lambda,\mu)})(g_2^{-1} x g_2) = f(A^{(\lambda,\mu)})(f(g_1^{-1} x g_1)) = f(A^{(\lambda,\mu)})(y')$,since f is a homomorphism, where $y' = f(g_1^{-1} x g_1) = g_2^{-1} y g_2 = \min\{A^{(\lambda,\mu)}(x') : f(x') = y' \, for \, x' \in G_1\} = \min\{A^{(\lambda,\mu)}(x') : f(g_1^{-1} x g_1) \, for \, x' \in G_1\} = \min\{ A^{(\lambda,\mu)}(g_1^{-1} x g_1) : f(g_1^{-1} x g_1) = y'\} = g_2^{-1} y g_2 \, for \, x \in A^{(\lambda,\mu)}, g_1 \in G_1\} = \min\{ A^{(\lambda,\mu)}(x) : f(g_1^{-1} x g_1) = y'\} = g_2^{-1} y g_2 \, for \, x \in A^{(\lambda,\mu)}, g_1 \in G_1\} = \min\{ A^{(\lambda,\mu)}(x) : f(g_1)^{-1} f(x) f(g_1) = g_2^{-1} y g_2 \, for \, x \in A^{(\lambda,\mu)}, g_1 \in G_1\} = \min\{ A^{(\lambda,\mu)}(x) : g_2^{-1} f(x) g_2 = g_2^{-1} y g_2 \, for \, x \in G_1\} = \min\{A^{(\lambda,\mu)}(x) : f(x) = y \, for \, x \in G_1\} = f(A^{(\lambda,\mu)})(y)$. Hence $({}^{(\lambda,\mu)})$ is a $(\lambda,\mu) - MAFNSG$ of G_2.

CONCLUSION

In the theory of fuzzy sets, the level subsets are vital role for its development. Similarly, the (λ, μ) − mutli fuzzy subgroups are very important role for the development of the theory of multi fuzzy subgroup of a group. In this paper an attempt has been made to study some algebraic natures of (λ, μ) − mutli anti fuzzy subgroups.

REFERENCES

1. Atanassov, K.T. (1986). Intuitionistic fuzzy sets. *Fuzzy sets and fuzzy systems, 20*(1), 87-96. https://doi.org/10.1016/S0165-0114(86)80034-3
2. Atanassov, K.T.(1994). New operation defined Over the intuitionistic fuzzy sets. *Fuzzy sets and fuzzy systems, 61*(2), 137-142. https://doi.org/10.1016/0165-0114(94)90229-1
3. Balasubramanian, K.R., Revathy, R and Rajangam,.R.(2021). (λ, μ)-multi fuzzy subgroups of a group. *Turkish Journal of computer and Mathematical Education*, *12*(11), 6148 -6160.
4. Basnet, D.K. & Sarma, N.K.(2010). A note on Intuitionistic Fuzzy Equivalence Relation. *International Mathematical Forum, 5*(67), 3301-3307.
5. Biswas, R.(2006). Vague Groups. *International Journal of Computational Cognition, 4*(2), 20-23.
6. Feng, Y. and Yao, B.(2012). On (λ, μ)-anti-fuzzy subgroups. *Feng and Yao Journal of Inequalities and Applications*, *2012*, 78. https://doi.org/10.1186/1029-242X-2012-78
7. Goguen, J.A.(1967). L-fuzzy set. *Journal of Mathematical analysis and Applications*, *18*, 145-174. https://doi.org/10.1016/0022-247X(67)90189-8
8. Mukharjee, N. P. and Bhattacharya, P.(1984). Fuzzy Normal Subgroups and Fuzzy Cosets. *Information Sciences, 34*(3), 225-239. https://doi.org/10.1016/0020-0255(84)90050-1
9. Muthuraj, R. and Balamurugan, S.(2013). Multi Anti Fuzzy Group and its Lower level Subgroup. *Gen. Math. Notes*, *17*(1), 74-81.
10. Muthuraj, R. and Balamurugan, S.(2014). A Study on Intuitionistic Multi Anti Fuzzy Subgroups. *Applied Mathematics and Sciences: An International Journal*, *1*(2).
11. Rosenfeld, A.(1971). Fuzzy Group. *Journal Of Mathematical Analysis and Applications*, *3*, 12-17.
12. Sabu, S. and Ramakrishnan, T.V.(2011a). Multi-Fuzzy Topology. *International Journal of Applied Mathematics*, *24*(1), 117-129.
13. Sabu, S., Ramakrishnan, T.V.(2010). Multi Fuzzy Sets. *International Mathematical Forum*, *50*, 2471-2476.
14. Sabu, S., Ramakrishnan, T.V.(2011b). Multi Fuzzy SubGroup. *Int. J. Contemp. Math. Sciences*, *6*(8),365-372.
15. Shen, Z.(1995). The anti-fuzzy subgroup of a group. *J Liaoning Normat Univ (Nat Sci), 18*(2), 99–101.
16. Sinoj, T.K. and Sunil, J.J.(2013). Intuitionistic Fuzzy Multi-Sets. *International Journal of Engineering Sciences and Innovaive Technology*, *2*(6),1-24.
17. Yao, B.(2005). (λ, μ)-fuzzy normal subgroups and (λ, μ)-fuzzy quotient subgroups. *Journal of Fuzzy Mathematics*, *13*(3), 695 – 705.
18. Yuan, X., Zhang, C., and Ren, Y.(2003). Generalized fuzzy groups and many-valued implications. *Fuzzy Sets Syst.*, *138*, 205–211. https://doi.org/10.1016/S0165-0114(02)00443-8
19. Zadeh, L.A.(1965). Fuzzy set. *Information and Control*, *8*, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X