

A Statistical Evolutionary Analysis and Prediction of Carbon-dioxide Emission in Gujarat, Maharashtra and Madhya Pradesh of India

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Keywords

Carbon-dioxide, Least Square Method, Coefficient of Determination, Residual Analysis, Regression Sum of Square, Residual Sum of Square, Green House Gases.

Article History

Received on 28th March 2023

Accepted on 5th April 2022

Published on 7th April 2023

Cite this article

Dubey, O. P., Lal, A. K., Mondal, K., & Singh, U. (2023). A Statistical Evolutionary Analysis and Prediction of Carbon-dioxide Emission in Gujarat, Maharashtra and Madhya Pradesh of India. *International Journal of Students' Research in Technology & Management*, 11(2), 01-08.
<https://doi.org/10.18510/ijstrtm.2023.1121>

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Abstract

Purpose of the study: The change of climate due to global warming has become a burning problem throughout the world. India is a country where global warming causes a concern to the scientists. Unrestricted emission of different green house gases mainly carbon dioxide is responsible for this alarming situation. This paper develops a state-wise nonlinear emission model of carbon dioxide, an important green house gas.

Methodology: Non linear least-square and regression analysis method used to explain the emission of the gas.

Main Findings: The short term and long-term forecast of carbon dioxide emission trend in the states are presented.

Applications of this study: Model is used in the states of Gujarat, Maharashtra and Madhya Pradesh of India.

Novelty/Originality of this study: Future prediction of emission of CO₂ may be evaluated from our proposed model. This paper may be helpful for the future researchers for finding emission of CO₂ and other GHGs for the remaining states in India.

INTRODUCTION

Global warming has reached at an alarming stage throughout the world. As a result we see the abrupt change of climate and weather. It is not a local issue now, it is a global phenomenon and the scientists along with the environmentalists are seriously worried about the evil effects of the unnatural change of weather. Polar ice sheets and glaciers are melting very fast resulting the rise of sea level (Khan, Z. A. (2017)). All on a sudden we often observe very extra weather events like tsunami, typhoons, droughts, heat wave etc. (Khan, A. A. (2018)).

This unnatural and unpredictable diversity of weather are caused due to indiscriminate human activities. Uncontrolled use of pesticides in agricultural field, unplanned set up of industry, indiscriminate deforestation, abuses of natural resources generates huge amount of Green House Gases (GHG) which gives result the high rise of temperature of the Earth known as Global Warming (Basak, P. and Nandi, S. (2015)). Among the GHG, carbon dioxide (CO₂) is mainly responsible for the high rise of temperature of the Earth (Battle, M., et al. (2010), Ghoshal, T. and Bhattacharyya, R. (2008)). In augmentation of CO₂ concentration in atmosphere, India is one of the leading countries in the world. Since India is a developing country, so there occur industrials set up in a rapid and unplanned way. Moreover, there are vast agricultural lands in India and as a result emissions of GHGs are occurring for burning of fossil fuels and land use changes and other industrial processes (Murthy, N. S., Panda, M. and Parikh, J. (1997a), Murthy, N. S., Panda, M. and Parikh, J. (1997b)). Due to degradation of O₃ layer, the heat wave radiates back down to the Earth which gives rise of temperature (Dewitte, S. and Clerbaux, N. (2018)). Moreover, burning of fossil fuels and emission from transport vehicles along with the rapid industrialization releases CO₂ and other GHGs that increases the temperature of atmosphere and it is being able to absorb more radiation and would warm up rapidly (Khansis, A. A. and Nettleman, M. D. (2005), Rufael, Y. W. (2010)). The main harmful gas which is responsible for global warming is CO₂ and that is why emission of CO₂ is the burning problem throughout the country. Few states of India are badly affected mainly for agricultural, industrial and population growth and increase emission of transportation. While some states are comparatively less emitter and less affected. Hence, it is important to study about the nature of emission of CO₂ in different states in India (Kram, T., et al.(2000)). Several research works has done about CO₂ emission (Basak, P. and Nandi, S. (2014), Ghoshal, T. and Bhattacharyya, R. (2008), Mondal, K., Basak, P. and Sinha, S. (2019a). in India. Also the emission is studied for different states of India (Mondal, K., Basak, P. and Sinha, S. (2019b)). In this article, the pattern of emission in the states of Gujarat, Maharashtra and Madhya Pradesh are studied and we would like develop a mathematical model to find the trend of CO₂ emission.

METHODOLOGY

Method of Least Square

The data of CO₂ emission for the 21 years (1980-2000) is utilized for the purpose of modeling. We formulated a third degree polynomial model for the analysis of CO₂ emission in the states of Gujarat, Maharashtra and Madhya Pradesh (Jin, R., et al.(2010)). For generating the model of CO₂ emission, we followed the works of (Basak, P. and Nandi, S. (2014), Tokos, C. P. and Xu, Y. (2009). Authors suggested a third degree polynomial model for emission of CO₂. For example:

$$Y = \alpha + \beta x + \gamma x^2 + \delta x^3 \quad \text{..... (1)}$$

Where Y is emission of carbon dioxide in '000 Metric ton of carbon and x represents time in years.

With the help of the given data $\{(x_1, y_1), (x_2, y_2), \dots \dots (x_n, y_n)\}$ we may define the error associated by,

$$v(\alpha, \beta, \gamma, \delta, \mu) = \sum_{i=1}^n (y_i - \alpha - \beta x_i - \gamma x_i^2 - \delta x_i^3)^2 \quad \text{..... (2)}$$

is a function of five variables α, β, γ and δ For minimizing the error and to estimate corresponding α, β, γ and δ multivariate calculus is used to have,

$$\frac{\partial v}{\partial \alpha} = 0, \frac{\partial v}{\partial \beta} = 0, \frac{\partial v}{\partial \gamma} = 0, \frac{\partial v}{\partial \delta} = 0,$$

Now corresponding normal equations are:

$$\begin{aligned} \sum y_i &= n\alpha + \beta \sum x_i + \gamma \sum x_i^2 + \delta \sum x_i^3 \\ \sum y_i x_i &= \alpha \sum x_i + \beta \sum x_i^2 + \gamma \sum x_i^3 + \delta \sum x_i^4 \\ \sum x_i^2 y_i &= \alpha \sum x_i^2 + \beta \sum x_i^3 + \gamma \sum x_i^4 + \delta \sum x_i^5 \\ \sum x_i^3 y_i &= \alpha \sum x_i^3 + \beta \sum x_i^4 + \gamma \sum x_i^5 + \delta \sum x_i^6 \end{aligned} \quad \text{..... (3)}$$

For given set of points $(x_i, y_i); (i=1, 2, \dots, n)$, the equations can be solved for α, β, γ and δ to find estimated $\alpha^{\#}, \beta^{\#}, \gamma^{\#}$ and $\delta^{\#}$ It has been found that in all the cases, the value of the 2nd order derivatives evolves to be positive at the points α, β, γ and δ . These satisfy the minimization criteria of v .

The third degree fitted polynomial of CO₂ emission is now estimated as,

$$Y = \alpha^{\#} + \beta^{\#} x + \gamma^{\#} x^2 + \delta^{\#} x^3 \quad \text{..... (4)}$$

Instantaneous Rate of Change of emission

Instantaneous Rate of Change (IROC) is a useful parameter that may be utilized to predict future emission of gas. For the computation of rate of change of the gas, the derivative of equation (4) is presented as,

$$\frac{dY}{dx} = \beta^{\#} + \gamma^{\#} x + \delta^{\#} x^2 \quad \text{..... (5)}$$

The equation (5) at a particular time is utilized for prediction of CO₂.

Quality of estimates

Equation (4) may be used for obtaining the estimation of the CO₂ emission for short and medium terms of time in a state. It remains to examine the goodness of fit for this estimation. The matter depends on the quality of the developed analytical models using the raw data. The quality of the proposed analytical models is verified with the statistical criteria, namely the coefficient of determination, adjusted coefficient of determination and residual analysis.

Coefficient of determination

The coefficient of determination (R^2) is defined as the proportion of the total response variation that is explained by the model. It provides an overall measure of how well the model fits. The general definition of R^2 is defined as,

$$R^2 = 1 - \frac{SS_{err}}{SS_{tot}}$$

Where, $SS_{tot} = \sum_i (y_i - \bar{y})^2$, $SS_{reg} = \sum_i (f_i - \bar{y})^2$, $SS_{err} = \sum_i (y_i - f_i)^2$

Here, SS_{tot} = Total sum of square (proportional to the sample variance), SS_{reg} = the regression sum of squares or the explained sum of square, and SS_{err} = the sum of squares of residuals, also called the residual sum of square. y_i and f_i are observed and estimated values of CO₂ emission.

Adjusted Coefficient of Determination

The Adjusted Coefficient of Determination (Adj R^2) is defined as,

$$R^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1} = R^2 - (1 - R^2) \frac{p}{n-p-1}$$

Where, p is the total number of regressors in the model (not counting the constant term) and n is the sample size. It is, however, another advanced measure how good the model fit the actual data.

RESULTS / FINDINGS

Gujarat

Using least square method, the CO₂ emission model for the state of Gujarat can be represent as,

$$Y(x) = -2.40393 \times 10^9 + 3.5957 \times 10^6 x - 1792.92x^2 + 0.298029x^3$$

Where, x represents time in year.

A graphical display of the observed data and estimated data for Gujarat is given in the Figure 1.

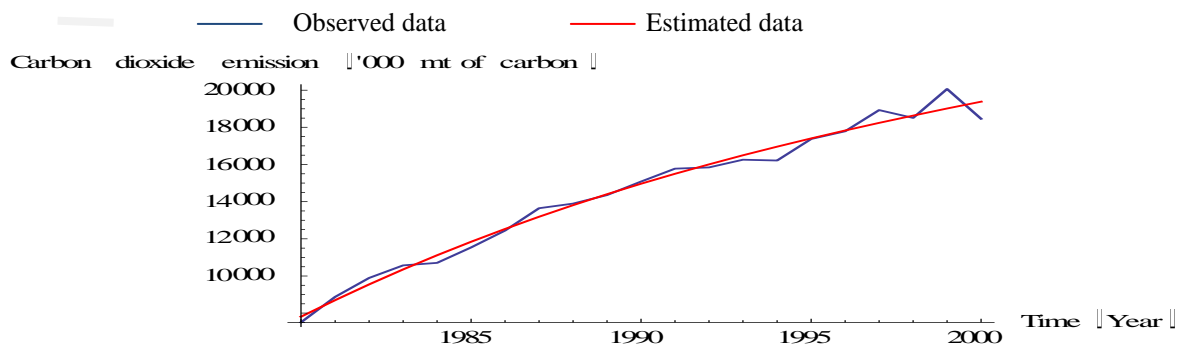


Figure 1: Emission of CO₂ in Gujarat.

From Figure 1, we can find out that CO₂ emission model for the state of Gujarat matches well with the actual status of CO₂ emission excepting a few years in the last lap. The model equation may be considered for obtaining the emission of CO₂ in Gujarat for short or medium term of time. It is necessary to measure the goodness of fit of the proposed model which can be obtained through the following analysis (Nandi, S. and Basak, P. (2014), Gokran, S. and Parikh, J. (1993)).

The values of R^2 (Adj R^2) are obtained and presented as:

Table 1: Statistical evaluation criteria for Gujarat

R square	Adjusted R square	FVU (Frac. Var. Unexplained)
0.9845	0.9818	0.0154
SS_{tot}	SS_{reg}	SS_{err}
257047595	253072270	3975324

A residual analysis is calculated of the proposed model of emission of CO₂ for the state of Gujarat is given as:

Table 2: Residual Analysis of CO₂ of Gujarat state

Year	Observed data (^{'000} MT of Carbon)	Estimated data (^{'000} MT of Carbon)	Residual	Standard Deviation of residual	Mean of residual	Standard Error of residual
1980	7501.63	7809.68	-308.054	445.832	0.0000055	97.2885
1981	8875.51	8699.37	176.136			
1982	9894.43	9545.59	348.836			
1983	10570.49	10350.1	220.36			
1984	10705.5	11114.8	-409.272			
1985	11536.44	11841.3	-304.868			
1986	12442.96	12531.5	-88.5646			
1987	13644.8	13187.2	457.588			
1988	13893.88	13810.2	83.7234			
1989	14355.4	14402.1	-46.7477			
1990	15081.9	14965	116.927			
1991	15776.39	15500.4	275.969			
1992	15840.44	16010.3	-169.84			
1993	16255.28	16496.3	-241.058			
1994	16214.27	16960.4	-746.113			
1995	17375.52	17404.2	-28.6828			
1996	17786.82	17829.6	-42.7662			
1997	18932.99	18238.3	694.669			
1998	18511.16	18632.2	-121.036			

1999	20066.85	19013	1053.85
2000	18461.46	19382.5	-921.058

According to the Table 2, the residuals are very small compared to data and the so is standard error. The results of Table 2 suggest a good quality of model in Gujarat.

Prediction of emission in Gujarat State

IROC of CO₂ emission for the state of Gujarat can be represent by the differential equation

$$\frac{dY}{dx}(\text{Gujarat}) = 3.5957 \times 10^6 - 3585.84x + 0.894087x^2 \quad \dots\dots\dots (6)$$

The graphical visualization of the IROC is presented in Figure 2.

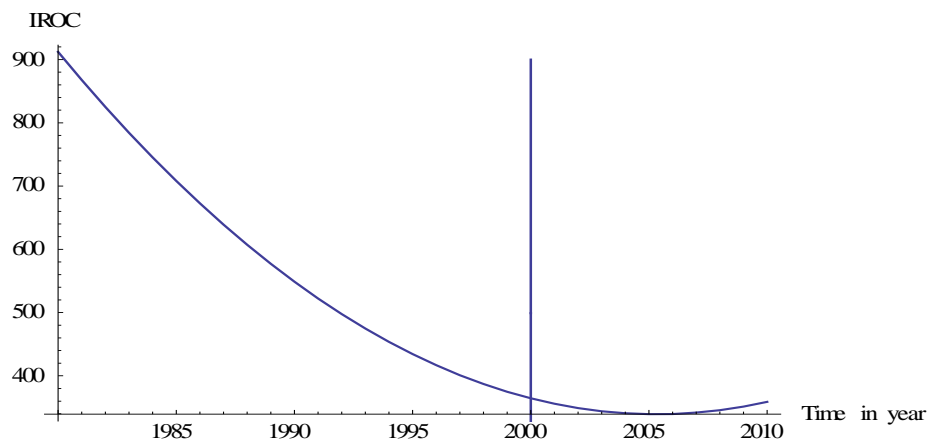


Figure 2: IROC of Gujarat

Equation (6) or the graph may be utilized to estimate the rate of change of CO₂ emission in Gujarat, for short or long term of time. We get total CO₂ emission in Gujarat in 2005, 2009 and 2012 are 21123.4, 22495.5 and 23587.5 ('000 Mt of carbon) respectively.

Maharashtra

The CO₂ emission model for the state of Maharashtra can be represented as,

$$Y(x) = 3.59853 \times 10^{10} - 5.42562 \times 10^7 x + 27267.2x^2 - 4.56767x^3$$

The observed data and model data can be represented graphically as below,

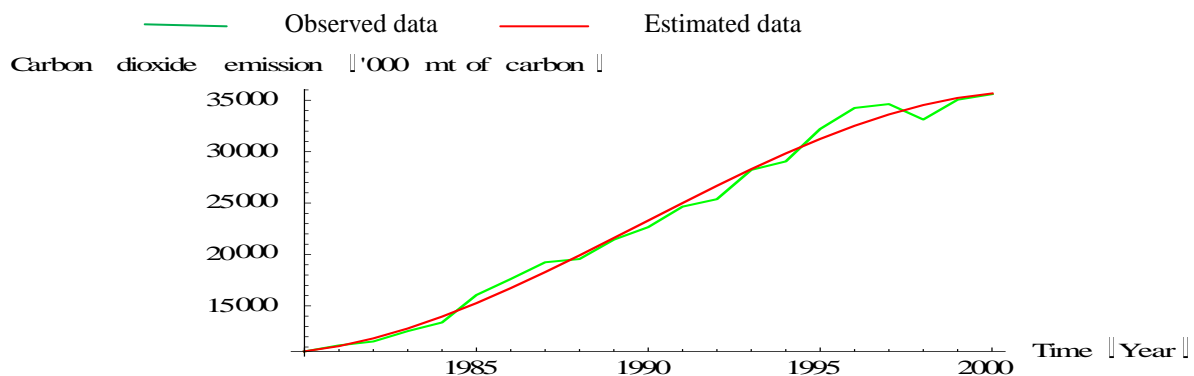


Figure 3: Emission of CO₂ in Maharashtra

According to the Figure 3, observed status of CO₂ emission and the data evaluated by the model matches almost perfectly. The statistical evaluation is given below in Table 3. For verifying the goodness of fit of the model SS_{tot} , SS_{reg} , SS_{err} and R^2 (Adj R^2) calculated. The value of R square is 0.99178 which is quite satisfactory and ensures that 99% of the total variation is extracted by the model.

Table 3: Statistical evaluation criteria for Maharashtra

R square	R square adjusted	FVU (Frac.Var. Unexplained)
0.9917	0.9903	0.0082
SS_{tot}	SS_{reg}	SS_{err}
1557968449	1545161988	12806459

Residual analysis is computed and the result is presented in the following Table 4. It is observed that residuals and standard errors of the residuals are considerably small compared to observed emission.

Table 4: Residual Analysis of CO₂ emission in Maharashtra

Year	Observed data (^{'000} MT of carbon)	Estimated data (^{'000} MT of Carbon)	Residual	Standard Deviation of residual	Mean of residual	Standard Error of residual
1980	10564.23	10575.7	-11.441	800.202	0.00	174.618
1981	11166.82	11084.2	82.5841			
1982	11534.16	11835.7	-301.582			
1983	12544.69	12802.8	-258.093			
1984	13382.98	13958	-574.973			
1985	16059.59	15273.8	785.744			
1986	17613.07	16723.1	890.014			
1987	19243.15	18278.2	964.973			
1988	19564.25	19911.8	-347.553			
1989	21449.69	21596.5	-146.837			
1990	22660.12	23304.9	-644.825			
1991	24651.46	25009.6	-358.189			
1992	25387.39	26683.2	-1295.84			
1993	28229.36	28298.3	-68.9329			
1994	29056.75	29827.4	-770.671			
1995	32216.88	31243.2	973.668			
1996	34255.33	32518.3	1737.07			
1997	34626.23	33625.2	1001.07			
1998	33120.79	34536.5	-1415.71			
1999	35051.86	35224.9	-173.019			
2000	35595.43	35662.9	-67.4623			

The result of statistical criteria and Residual analysis suggest that proposed model may be used as future prediction of emissions of CO₂ for the state of Maharashtra.

Prediction of emission in Maharashtra State

The IROC of CO₂ emission for the states of Maharashtra can be represent by the differential equation,

$$\frac{dy}{dx}(\text{Maharashtra}) = -5.42562 \times 10^7 + 54534.3x - 13.703x^2 \quad \dots\dots\dots (7)$$

The graphical display of expression (7) is given by,

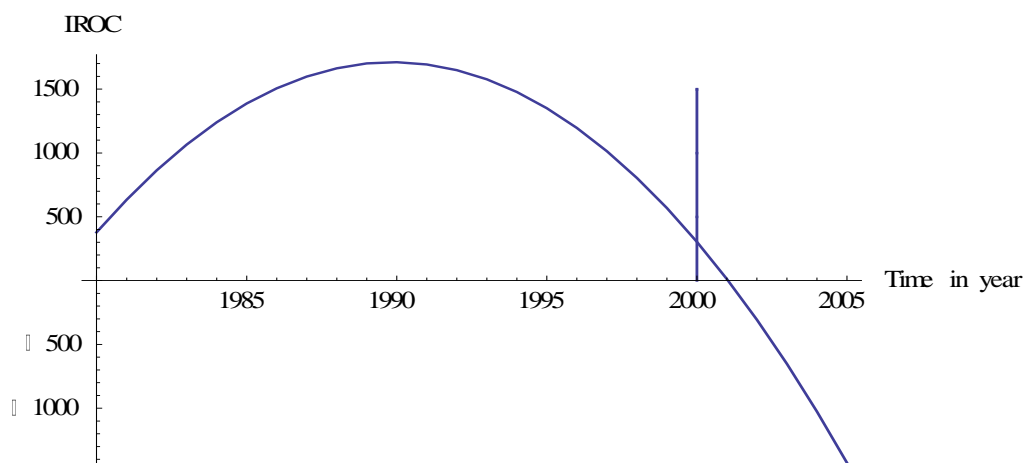


Figure 4: IROC of Maharashtra

Equation (7) or Fig. 4 may be utilized to estimate the rate of change of CO₂ emission in Maharashtra for short or long term of time. Total CO₂ emission in the state in 2005, 2009 and 2012 are 33138.2, 23816.5 and 11414.6 (^{'000} Mt of carbon) respectively. IROC shows rapid rate of decrease towards the end of the data.

Madhya Pradesh

The model for the CO₂ emission of Madhya Pradesh can be represent as,

$$Y(x) = 3.91803 \times 10^{11} - 5.91014 \times 10^7 x + 29716.2x^2 - 4.98026x^3$$

A graphical display of the actual data and model data for Madhya Pradesh is given by Figure 5.

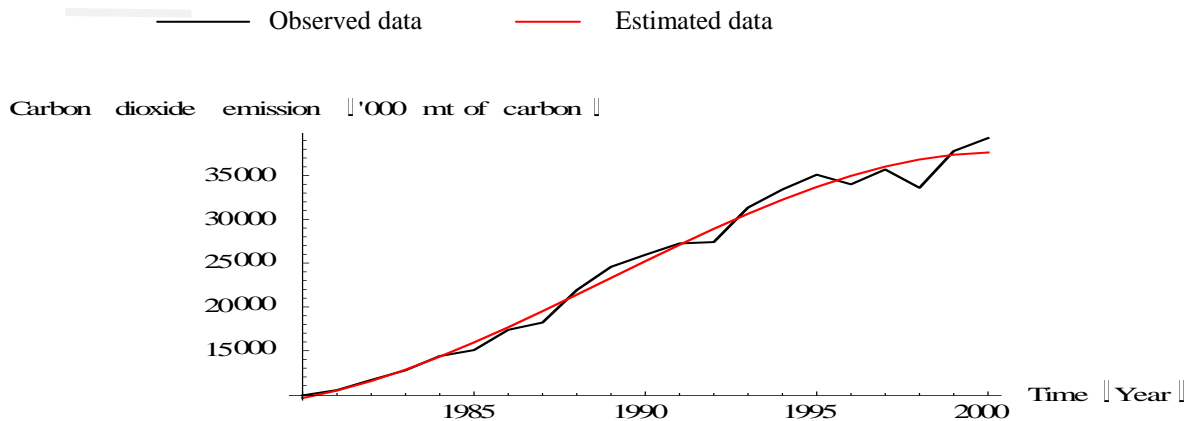


Figure 5: Emission of CO₂ in Madhya Pradesh

Above model may be used to get estimation of CO₂ emission for the state of Madhya Pradesh for short or medium terms of time. The R square and R square (adjusted) are also computed and the result indicates that a good model have been identified. The statistical criteria are displaced in Table 5.

Table 5: Statistical evaluation criteria of Madhya Pradesh

R Square	R square adjusted	FVU (Frac. Var. Unexplained)
0.9868	0.9845	0.0131
SS_{tot}	SS_{reg}	SS_{err}
1949980722	1924359560	25621161

Finally, the residual analysis is computed on the proposed model of CO₂ emission for the state of Madhya Pradesh. From the Table 6 it is clear that the residuals and standard error of residuals are too small compared to the data. It endorses that a good model is identified.

Table 6: Residual Analysis of CO₂ emission in Madhya Pradesh

Year	Observed data (⁰⁰⁰ MT of Carbon)	Empirical data (⁰⁰⁰ MT of carbon)	Residual	Standard Deviation of residual	Mean of Residual	Standard Error of residual
1980	9890.58	9593.81	296.77	1131.84	0.00476	246.988
1981	10501.97	10448.2	53.77			
1982	11656.09	11539.5	116.59			
1983	12753.7	12837.8	-84.10			
1984	14390.89	14313.3	77.59			
1985	15065.49	15936.1	-870.61			
1986	17388.37	17676.2	-287.83			
1987	18206.35	19503.9	-1297.55			
1988	21916.93	21389.2	527.73			
1989	24574.08	23302.2	1271.88			
1990	25945.04	25213	732.04			
1991	27238.76	27091.9	146.86			
1992	27406.21	28908.8	-1502.59			
1993	31345.35	30634	711.35			
1994	33412.05	32237.5	1174.55			
1995	35096.14	33689.5	1406.64			
1996	33988.91	34960	-971.09			
1997	35698.32	36019.2	-320.88			
1998	33601.08	36837.2	-3236.12			
1999	37790.15	37384.2	405.95			
2000	39279.35	37630.2	1649.15			

Prediction of emission in Madhya Pradesh State

The IROC of CO₂ emission for the state of Madhya Pradesh can be represent by the differential equation,

$$\frac{dy}{dx}(\text{Madhya Pradesh}) = -5.91014 \times 10^7 + 59432.3x - 14.9408x^2 \quad \dots\dots\dots (8)$$

The expression (8) can expressed visually as ,

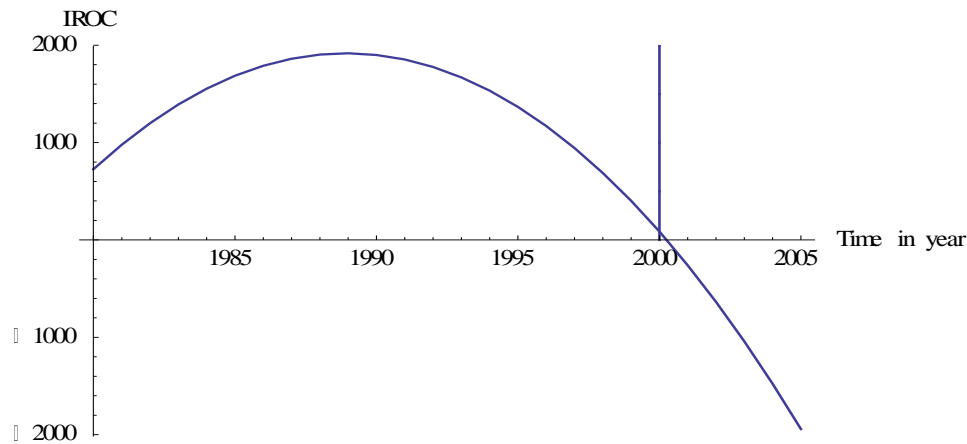


Figure 6: IROC of Madhya Pradesh

ANALYSIS

Considering the proposed model, the future emission of CO₂ is evaluated and shows a rapid decreasing rate of change at the end of the curve and total CO₂ estimated emission in Madhya Pradesh in 2005, 2009 and 2012 are 33299.9, 21370.6 and 6230.66 ('000 Mt of carbon) respectively.

CONCLUSION

Regarding the prediction, for Gujarat state IROC graph is strictly decreasing, whereas for the states of Maharashtra and Madhya Pradesh it is increasing up to the year of 1990 and then it gradually decreases. Increasing trend of IROC indicates that uncontrolled emission of CO₂ from different sources. Certainly, it indicates that in Gujarat, the uncontrolled emission still persists and is somewhat controlled as a whole in the states of Madhya Pradesh and Maharashtra. In this article, a third degree polynomial has been developed as a CO₂ emission model utilizing the observed data of the emission of CO₂ of about 21 years and using the method of least square that characterize well the emission pattern for the states of Gujarat, Maharashtra and Madhya Pradesh. Besides, computing total emission of the gas for the subjected states, some statistical procedure applied to clarify the goodness of fit of the proposed model namely, coefficient of determination R², R² (Adjusted) and residual analysis. The numerical value of the statistical procedure reflects the fact that we have chosen a good model. The value of R² for the states Gujarat and Madhya Pradesh is 0.9845 and 0.9868 respectively where that is for Maharashtra is 0.9903. These numerical values indicate that proposed model is fitted fairly well. Future prediction of emission of CO₂ may be evaluated from our proposed model. This paper may be helpful for the future researchers for finding emission of CO₂ and other GHGs for the remaining states in India.

ACKNOWLEDGEMENT

We would like to thank Prof. K. C. Prasad, Department of Mathematics, Ranchi University, Ranchi, India and all those who helped us gather the information needed for this research work.

AUTHORS CONTRIBUTION

The authors confirm contribution to the manuscript as follows: study conception and design: Kartick Mondal, data analysis: Umashankar Singh; analysis and interpretation of results: Om Prakash Dubey and Anmol Kumar Lal, draft manuscript preparation: Om Prakash Dubey.

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